AD-A091 016 WYLE LABS EL SEGUNDO CA F/G 20/1

CORRECTION PROCEDURES FOR AIRCRAFT NOISE DATA. VOLUME III. FILT--ETC(U)

JUL 80 L C SUTHERLAND DOT-FA78WA-4143

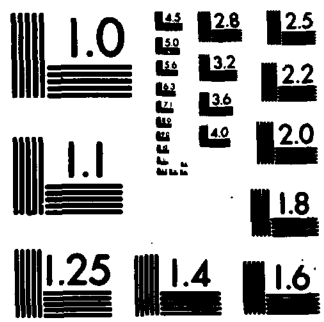
UNCLASSIFIED WR-79-9-VOL-3 FAA/EE-80-1-VOL-3 NL

END

DATE FILMED

1-8

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Report No: FAA-EE-80-1, Vol. 3

LEVEL III (12)

A083 075

AD A091016

Correction Procedures for Aircraft Noise Data Volume 3: Filter Effects

Louis C. Sutherland
Wyle Laboratories
128 Maryland St.
El Segundo, CA. 90245

DTIC
ELECTE
OCT 29 1980
S D
A

July 1980
Final Report

Document is available to the U.S. public through
The National Technical Information Service,
Springfield, Virginia 22161

Prepared for

U.S. DEPARTMENT OF TRANSPORTATION
Federal Aviation Administration
Office of Environment and Energy
Washington, D.C. 20591

DDC FILE COPY

80 10 27 010

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
18 19 FAA/EE/80-1 - VOL-3 ~~III~~	AD-A091016	

4. Title and Subtitle	
6 Correction Procedures for Aircraft Noise Data. Volume III. ~~Volume III~~ Filter Effects.	5. Report Date
	6. Performing Organization Code
7. Author(s) 10 Louis C. Sutherland	8. Performing Organization Report No. 14 WR-79-9 - VOL-3 ~~III~~
9. Performing Organization Name and Address Wyle Laboratories Wyle Research 128 Maryland Street El Segundo, California 90245	10. Work Unit No. (TRAIS)
	11. Contract or Grant No. 15 DOT-FA 78 WA-4143
	13. Type of Report and Period Covered 9 Final rept.
12. Sponsoring Agency Name and Address U.S. Department of Transportation Federal Aviation Administration Office of Environment and Energy Washington, D.C. 20591	14. Sponsoring Agency Code

15. Supplementary Notes

Final 11 Jul 80 12 93

16. Abstract

A method has been developed to quantify errors in measurement, correction and extrapolation of aircraft noise data which are attributable to spectrum slope and nonideal filter transmission characteristics. The method's fundamental basis is an iterative calculation scheme which recovers an estimate of the true spectrum shape of a signal from its measured band levels. This "true" spectrum can then be extrapolated to other distances or have frequency dependent corrections made to it on an exact basis without need for worry about filter or slope errors. The new spectrum may then be re-integrated to produce band levels suitable for input to EPNdB or PNL calculations. Applying this procedure to real flyover data and comparing the resultant values of EPNL with those using FAR Part 36 procedures showed only small differences usually. However, the technique provides a consistent method for the analysis of errors due to these sources and should be useful for estimating their magnitude in the future.

17. Key Words	18. Distribution Statement
	Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	91	

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

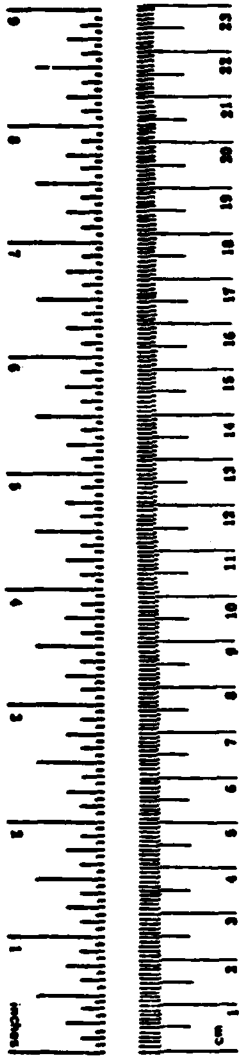
382150 JM

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

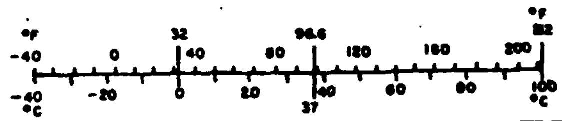


TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Codes

A

I. INTRODUCTORY SUMMARY

This report considers the errors attributable to the effects of non-ideal filter transmission characteristics (e.g., finite slopes for the filter skirts) and varying slopes of the spectrum applied to the filter input, upon the measurement, correction, or extrapolation (in the presence of masking by background noise) of aircraft noise data. This report is the third in a series of reports on correction procedures for the evaluation of aircraft noise.[1-4]

Current engineering practice for evaluation of the spectral content of aircraft noise is well defined in FAA Part 36,[5] and in a number of related standards and documents covering the details of aircraft noise measurement and spectrum analysis.[6-8] In current engineering practice, exemplified by the procedures cited in these references, no corrections are applied for filter sideband or spectrum slope effects when analyzing the aircraft spectrum data.

The basic approach utilized in this report for analyzing these errors due to the filter skirts and signal spectrum slopes involves, first, computing a close approximation to the true spectrum shape of the signal at all frequencies. This is one of the key departures from recent related studies which have focused on just one aspect of the problem - specifying attenuation of a band of noise due to atmospheric absorption, given a definition of attenuation at single frequencies.[9-15] The method outlined in this report is applicable to this and other problems encountered in evaluation or correction of aircraft spectra. The method is based on the use of a curve-fitting spline function to estimate the spectrum shape between center frequencies of three adjacent one-third octave bands. The method includes a critical iteration routine to ensure that the integration of this estimated spectrum over the effective transmission bandwidth of each one-third octave band filter produces the same band level as actually specified by the measured value. It is impossible, of course, to know the exact shape of the aircraft noise spectrum at all frequencies between the nominal band center frequencies, since only the integrated band level is specified by the measured data. However, the analytical methods employed here for estimating and verifying this spectrum shape provide one rational and consistent approach to the problem. Although the precise quantitative results obtained with this iterative technique are dependent upon the

exact form of the spectrum shape estimating procedure used herein (i.e., the numerical curve-fitting spline function and spectrum iteration routine), it can be expected that essentially the same result would be obtained with the use of any other spectrum interpolation schemes which can accurately account for curvature in the spectrum shape.

Once this general spectrum shape, corresponding to the initial levels measured with real filters, is estimated, corrections or adjustments for different filters (i.e., perfect filters) or different weather or propagation distances can then be applied to the spectrum at all frequencies and the corrected spectrum integrated again, for each filter band, to obtain new adjusted or corrected band levels. Based on the techniques outlined in this report, these corrected band levels, and, for aircraft signals, corresponding values of perceived or effective perceived noise level can be expressed in terms of values that would have been measured with an ideal filter or with a standardized version of a practical filter.

The report also outlines how one can estimate, within certain constraints, corrections to a measured spectrum to account for the influence of additive background noise in combination with the effects of finite filter slopes and extreme signal spectrum slopes.

Following this introduction, Section 2 defines the type and magnitude of anomalies that occur in the spectral analysis of aircraft noise due to the individual and combined effects of large negative slopes in the measured spectrum and finite slopes of the filter skirts with or without the presence of background noise and discusses alternative procedures which have or might be used for correcting spectra for filter and spectrum slope effects.

Section 3 defines the general iterative approach used to estimate the true spectrum shape and outlines the design of a computer algorithm based on this approach which can be used to correct measured band levels for the effects of finite filter skirts and extreme spectrum slopes. As a demonstration of this technique, it is applied to a real aircraft spectrum measured at one distance and weather condition and the one-third octave band levels from this same source that would be measured with the same analyzer are predicted for a new distance and weather condition. This new band level spectrum is then used as an input to the iterative algorithm and the original measured band levels are obtained very closely.

Section 4 presents the results of applying this approach to a variety of idealized and actual aircraft spectra to further demonstrate the relative accuracy of the technique and to illustrate the estimated differences between PNL or EPNL values obtained when this technique is used instead of current methods for specifying band levels and band level corrections. The former allows one to simulate and evaluate, explicitly, errors due to finite slopes of analysis filters and signal spectra; such errors are only implicit in current methods.

It is shown that under typical conditions specified in the report, these differences in PNL or EPNL values are usually less than 1 dB. However, when high frequency one-third octave band spectrum measurements are made, or corrected to, large distances under conditions defined roughly by the simple rule of thumb,

$$\text{Distance, in meters} \approx 10^5 / (\text{frequency, Hz})^2$$

band levels can be in error by much more than 1 dB.

Finally, Section 5 summarizes suggested procedures and algorithms for correcting measured aircraft noise spectra for the influence of finite filter and extreme signal spectrum slopes.

Appendices follow, which contain:

A. Simplified model for output of a finite filter for both signal and background input noise components.

B. Supporting details pertinent to this report in the form of a reprint, with errata, from a recently published paper in the Journal of the Acoustic Society of America.

C. Analysis of power transmission through a practical fractional-octave band filter.

2. ANOMALIES IN BAND LEVELS FROM SPECTRAL ANALYSIS OF BROADBAND NOISE

Three possible ways to define band levels, identified below and illustrated in Figure 1, might be used to interpret the spectral content of a broadband noise which is evaluated with a typical spectrum analyzer.

- The reference band level, L_{Bo}, for a reference white noise signal which has a constant spectrum level equal to that of the signal at the band center frequency f_o
- The ideal band level, L_{BI}, of the true spectrum as measured with a perfect filter over the nominal filter bandwidth f_1 to f_2.
- The measured band level, L_B, of the true spectrum as measured with a practical filter over the significant power transmission of the filter which is in the frequency range f_L to f_U.

The first form, L_{Bo}, is identified here since it is often implied as the effective band level when carrying out frequency variable corrections to signal spectra. For example, one way to express and compute the attenuation between two bands of noise due to any attenuation process whose magnitude varies with frequency, implies this format for expressing band levels when the attenuation can be accurately defined only at single frequencies, such as at the band center frequencies.

The second form, L_{BI}, would presumably be the desired form for universal application. It is, in fact, closely approximated in many situations when the signal spectrum slope is not large and spectrum analysis filters are employed which have very steep skirts.

The last form, L_B, represents the actual band level that is measured with real filters.

Each of these three forms can be described, mathematically, in the following fashion.

$$L_{Bo} = L_s(f_c) + 10 \log \left[f_2 - f_1 \right] \quad , \text{dB} \qquad (1)$$

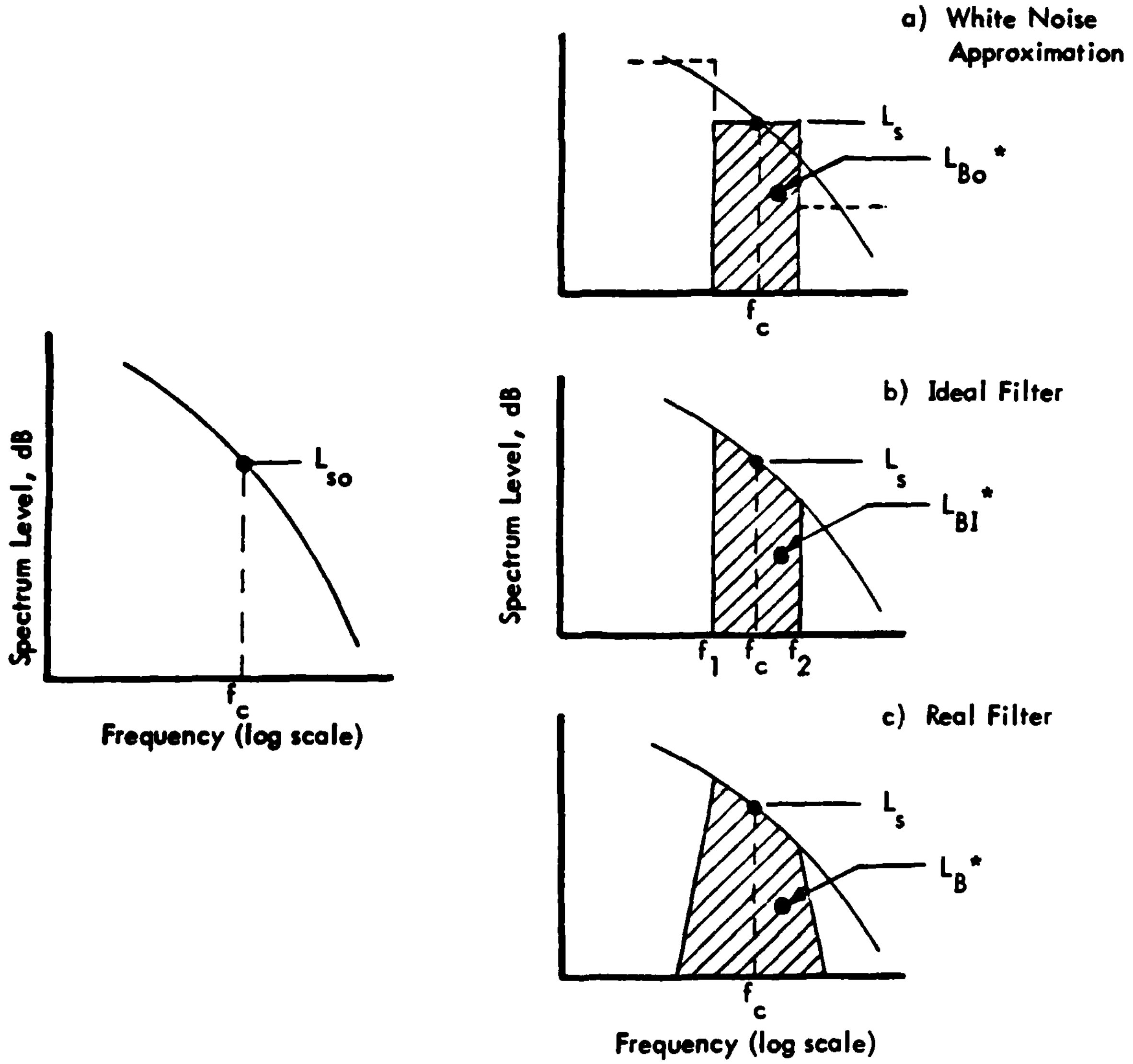


Figure 1. Conceptual Illustration of Three Alternative Ways to Interpret a Band Level Centered at Frequency f_c with a Spectrum Level L_s at this Frequency (*cross-hatched area equals indicated level)

$$L_{BI} = 10 \log_{10} \left[\int_{f_1}^{f_2} 10^{L_s(f)/10} \, df \right] \text{, dB} \quad (2)$$

$$L_B = 10 \log_{10} \left[\int_{f_L}^{f_U} 10^{\left[L_s(f) - TL(f)\right]/10} \, df \right] \text{, dB} \quad (3)$$

where

$L_s(f)$ = spectrum level at frequency f
$10 \log_{10} \left[P^2(f)/P_o^2\right]$

P_o = normal reference pressure = 20 μPa

f_1, f_c, f_2 = lower band edge, geometric center, and upper band edge frequency of an ideal filter

TL(f) = power transmission loss of real filter

f_L, f_U = lower and upper frequency limits of effective transmission range of filter $\simeq$ $f_1/5$ and $5 \cdot f_U$ respectively

This report will consider differences between these three forms for expressing band levels, first for single and then multiple bands constituting a complete spectrum. Finally, however, it will consider differences in the single-number, frequency-weighted and time-integrated metric for aircraft noise, Perceived Noise Level (PNL) and Effective Perceived Noise Level (EPNL) respectively when these are evaluated with either of the last two forms above for expressing band levels, namely, the ideal and practical band levels L_{BI} and L_B.

In developing this information, it will become clear that the first form for expressing band levels is also very useful, conceptually, as an intermediate transition medium for estimating changes in band levels due to any frequency-sensitive attenuation process. One such process is obviously atmospheric absorption, which plays such a strong role in shaping the time-varying spectrum of aircraft noise signatures during a flyover.

Define, now, two necessary and sufficient "error" terms which will constitute the building blocks upon which the principal results in this report will be developed. (For now, the compounding effects or errors in spectrum analysis due to background noise are ignored.) Let the reader not dispair at this point. These "error" terms are really used only to aid in understanding the basis for the results obtained. These results will be expressed in more practical ways which do not require keeping track of these error terms explicitly.

- Spectrum Slope Error, Δ_S

This will be defined as simply equal to the difference between the ideal and reference band levels, L_{BI}, and L_{Bo}, so that

$$\Delta_S = L_{BI} - L_{Bo} = L_{BI} - \left[L_S(f_{c_o}) + 10 \log_{10} (f_2 - f_1) \right] \quad , \text{dB} \tag{4a}$$

or, solving for the spectrum level, $L_S(f_o)$

$$L_S(f_{o_c}) = L_{BI} - 10 \log_{10} (f_2 - f_1) - \Delta_S \quad , \text{dB} \tag{4b}$$

As will be clear later, it is this second form in eq. (4b) which is more useful in developing a direct and unified approach to account for finite filter and signal spectrum slopes. Strictly speaking, this quantity, Δ_S, is not really an error in spectrum analysis but is so designated here for simplicity and consistency in terminology.

- Filter Error, Δ_F

This can be considered a real error in band levels and is defined as the difference between band levels measured with a real (L_B) and ideal (L_{BI}) filter, or

$$\Delta_F = L_B - L_{BI} \quad , \text{dB} \tag{5}$$

That is, Δ_F, is the difference between the band level measured with practical spectrum analysis filters having finite slopes of their filter skirts and the ideal band level that would be measured with a perfect filter with zero transmission outside of its nominal passband. This error is much less for one-third octave than for full octave band filters and is most significant at high frequencies and large distances from a source when the spectrum slope can be very steep.

The next section will outline one unified approach to aircraft spectral analysis which has the potential ability to account for both of these errors simultaneously. First, however, it is desirable to obtain a quantitative picture of the conditions under which these two types of errors occur and their magnitude under these conditions.

It will be shown that, in general, differences in weighted overall aircraft noise levels, expressed in terms of PNL or EPNL, will usually be small - seldom exceeding 1 to 2 dB and usually much less. However, when aircraft noise signatures are being analyzed for detailed evaluation of source contributions, large errors can occur in specific one-third octave band levels that can be quite important and deserve more careful consideration.

One example of how these "error" terms might be applied, conceptualy, is shown in Figure 2. This illustrates, schematically, one basic aspect of aircraft flyover spectrum evaluation - the prediction of how a given measured spectrum at one point - identified in Figure 2 as the "source" - changes as the sound propagates to another - the receiver. This common source-path-receiver calculation of propagation loss (and corresponding spectrum shaping) is applied, twice, when correcting data measured under one condition of weather and propagation path length to another condition with different weather and/or path length. The only difference is that the calculation starts at the receiver, works backward to the presumed source under test conditions and then back to the reference receiver for reference conditions.

As indicated near the bottom of Figure 2, it is current engineering practice to compute a receiver band level $L_B(R)$ directly from a source band level $L_B(s)$ by subtracting a single band attenuation ΔL_B due to atmospheric attenuation which depends only on the weather conditions and total path length.[16] (Spreading loss is ignored here, since it is not frequency-sensitive.) The alternative approach taken in this report makes it possible to account indirectly for the two analysis errors Δ_F and Δ_S, first at the source and then at the receiver, as illustrated in the figure. The problem, illustrated in Figure 2, is treated in detail in reference 14, included herein as Appendix B, but with one variation. The difference between Δ_S at the source and receiver was used to define a single correction factor relating the

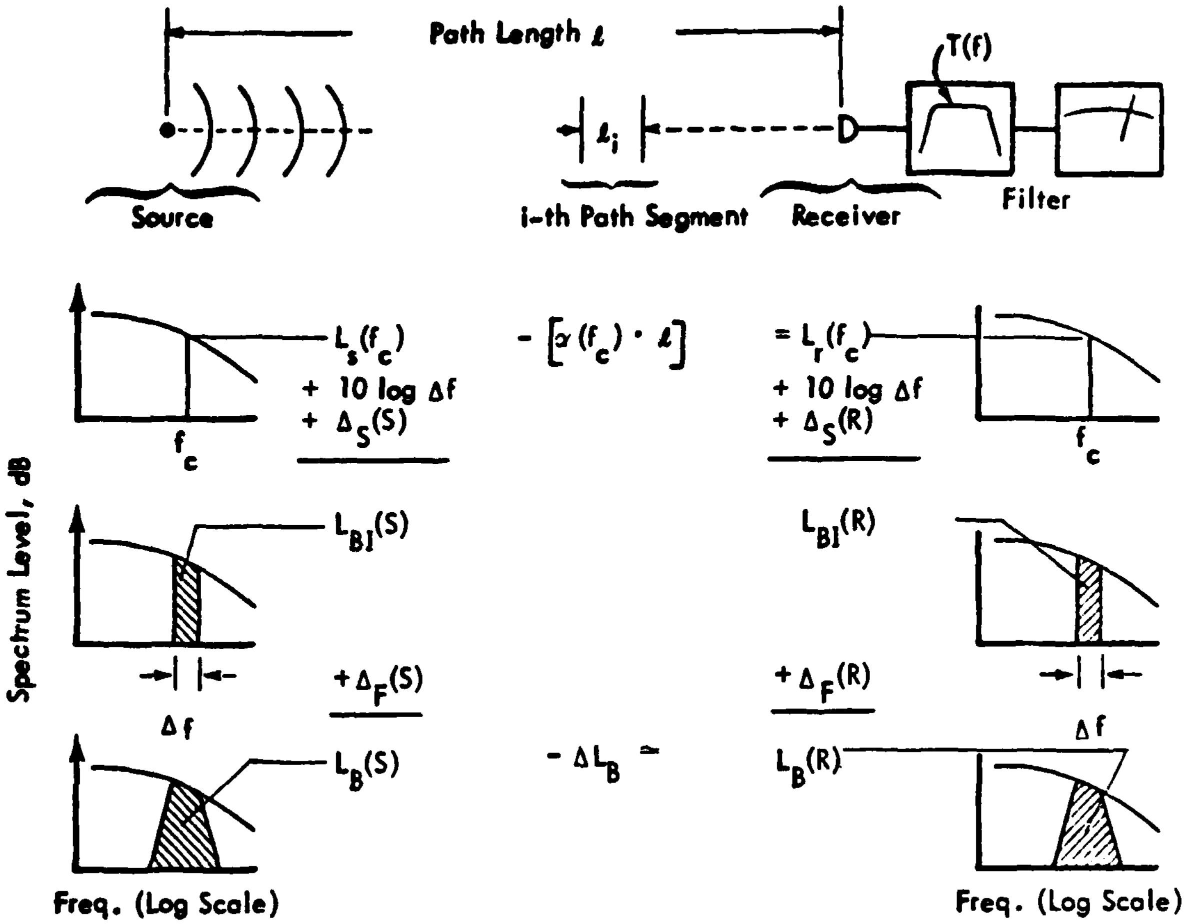


Figure 2. Illustration of the Conceptual Application of Filter and Spectrum Slope Errors Δ_F and Δ_S Respectively to Estimate the Propagation Loss Between a Measured Source Band Level $L_B(S)$ and an Estimated Receiver Band Level $L_B(R)$. In each case, the measured band level, L_B is corrected by Δ_F to obtain the ideal band level, L_{BI}, assuming a perfect filter. Then the spectrum slope error Δ_S is applied to L_{BI}, along with the bandwidth term 10 log Δ_F, to obtain the spectrum level at the band center frequency. The difference between the spectrum levels at the source and receiver is, ignoring spreading loss, simply the atmospheric attenuation $\alpha(f_c) \cdot \ell$ at the center frequency f_c over the path length ℓ where $\alpha(f_c)$ is the atmospheric absorption coefficient at this frequency.

difference between absorption loss for ideal bands of noise and the single frequency value $\alpha(f_c) \cdot \ell$ at the band center frequency.

It must be emphasized that the preceding discussion of methods for computing absorption loss for bands of noise is only provided here as an illustration of the conceptual application of the two "error" terms Δ_F and Δ_S. Nevertheless, it is appropriate to bring out the aspect of atmospheric absorption loss which has such a strong spectrum shaping influence on aircraft noise signatures. Specifically, the type of high frequency roll-off in aircraft noise propagated over long distances is, in fact, the principal basis for considering potential errors in aircraft single-event noise metrics, such as PNL or EPNL, due to the combined effect of these filter and spectrum slope effects. Consider, now, more specific quantitative aspects of these error terms.

2.1 Filter Error Due to Finite Filter Skirts

The well-known filter error, Δ_F, is inherent in the fundamental process of signal spectrum analysis and has been evaluated extensively by others including, for example, Beranek[17] and Sepmeyer.[18, 19] A detailed mathematical derivation of this inherent error in filter band levels is presented in Section III of Appendix B. The approach is based on the detailed treatment in Appendix C of the power transmission loss TL(f) term for the practical filter, included in eq. (3). This power transmission is simulated by defining, mathematically, a suitable expression for the filter response TL(f) which closely approximates the response of real filters. Then, by numerical integration of the power transmitted through this filter and an ideal rectangular filter with the same band center frequency, the filter correction factor Δ_f is computed from equations (2), (3), and (5).

Examples of this inherent error in spectral analysis will be illustrated later with hypothetical and actual aircraft spectra to show that it can produce substantial errors in specific one-third octave band levels, even close to a source, when the spectral shape is quite uneven. Clearly, any data evaluation procedure which makes it possible to account for this error directly could be very useful.

2.2 Spectrum Slope Error

As pointed out earlier, this more obscure "error" is, in effect, implicit when one applies corrections valid only at single frequencies to band level data. Recall that this error, defined by equations (1), (2), and (4a), is simply the difference between the band level of a spectrum measured with a perfect filter (L_{BI}) and the band level (L_{Bo}) that one would obtain if the spectral density over the entire band were the same as at the band center frequency.

Based on the mathematical details developed in Appendix B for the integrated power transmitted through an ideal filter band (see equations (4) to (6b), (10) and (11) on page B-3 of Appendix B), one can derive the following expression for Δ_S for the case of an input signal with a constant band level slope applied to an ideal filter.

$$\Delta_S = 10 \log_{10} \left[r^{1/2} \left(r^{(m+1)/2} - r^{-(m+1)/2} \right) \Big/ (m+1)(r-1) \right] \quad , \text{dB} \qquad (6a)$$

where

r = ratio of upper to lower band edge frequencies f_2 and f_1, respectively, for an ideal rectangular filter

$m+1$ = $S/10 \log 2$

and

S = band level slope of the true input spectrum in dB/octave

For the case of $m = -1$ or $S = 0$ dB/octave (pink noise),

$$\Delta_S = 10 \log_{10} \left[r^{1/2} \left(\ln_e r \right) (r-1) \right] \qquad (6b)$$

2.3 Filter and Slope Errors for Constant Slope Source Spectra

Utilizing the concepts just defined in the preceding two sections, it is desirable to examine these two spectrum analysis error terms for the case of a signal input representing a source signal with a constant band level slope, before spectrum shaping by atmospheric attenuation has occurred.

The results are shown in Figure 3. Part (a) shows the slope error $\Delta_S(S)$, as defined by eq. (4a), at the source for full, one-third and 1/18th octave band filters. The latter represent the effective spectral filtering bandwidth to be employed later. (Note the scale change for this case.)

Part (b) shows the filter error, as defined by eq. (5) for the full and one-third octave band filters. Note that this error is of the order of 1/2 of Δ_S for the range of slopes considered.

For most aircraft noise signals measured close to the source, say, within 75 m (~250 ft), the band level slope is usually less than 9 dB/octave unless pure tone components are present. In their absence, it is clear, from Figure 3, that both Δ_F and Δ_S would be very small, for one-third octave band filters. Under such conditions, the actual "as measured" band levels could be considered free of side-band effects due to the filter skirts and the spectral density at the band center can be closely approximated by the "white noise" approximation implicit in eq. (1). As we will see, this ideal situation does not hold when significant tones are added to the broadband signal.

2.4 Filter and Slope Errors at a Distant Receiver

Consider, now, the change in Δ_F and Δ_S at a receiver due to spectral shaping by atmospheric absorption. A variety of constant slope source spectra, with the addition of a pure tone component in one case, have been selected for this evaluation. These idealized source spectra are illustrated in Figure 4. The curves show source spectra with slopes ranging from +3 dB/octave to -36 dB/octave - a range sufficient to cover normal source spectra plus special cases involving, for example, pseudotones, where local spectral slopes at a source signal may be quite high.

It was assumed that the source spectra were measured with an ideal filter so that the initial spectral content at the signal could be determined exactly. Then, by propagating these known spectra, through known atmospheric absorption losses, one can redefine a new attenuated spectra at a receiver and then apply the band integration concept described at the beginning of this section and developed in detail in Appendix C. In this case, it was possible to compute new band levels at

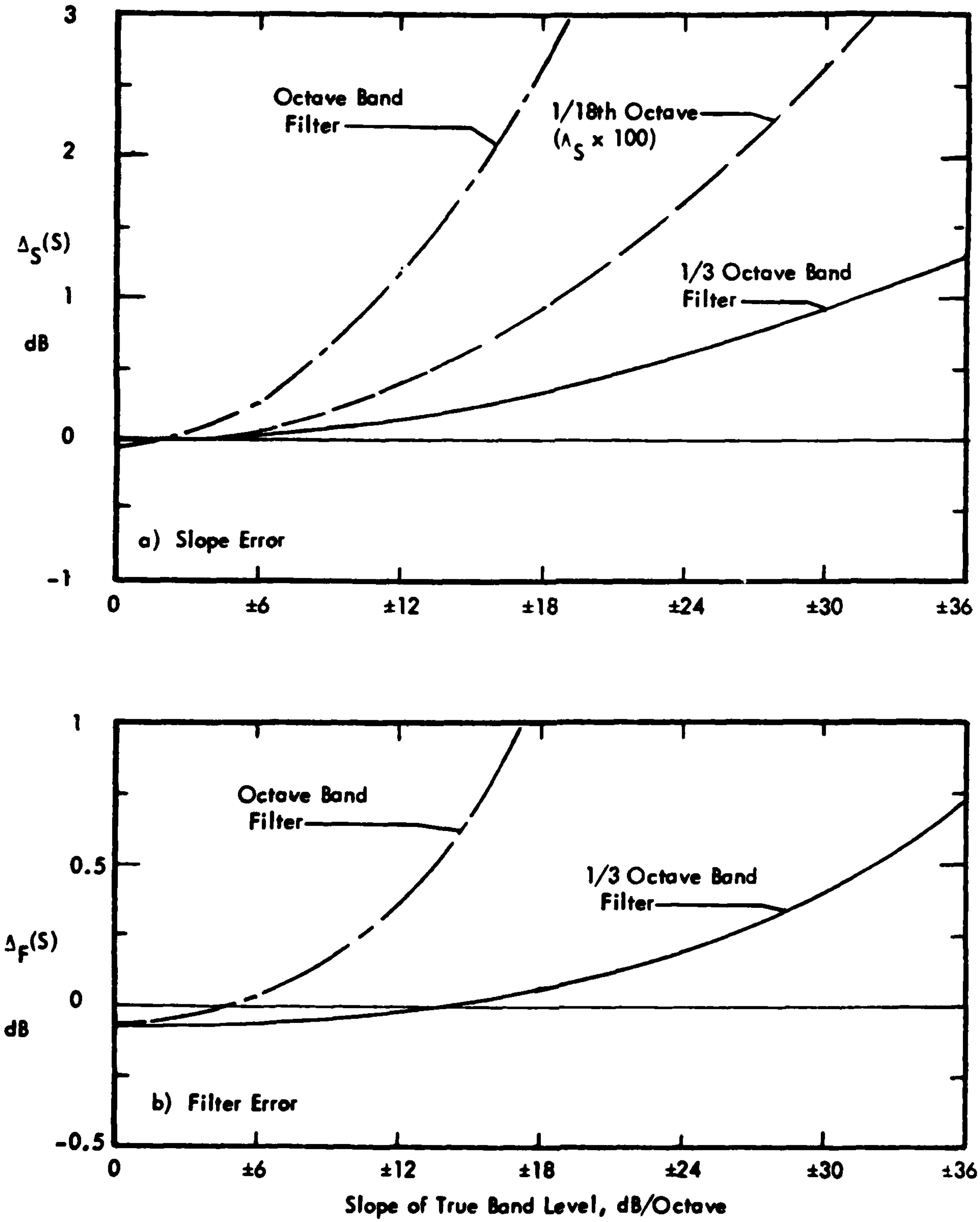


Figure 3. Slope and Filter Error for Constant Slope Source Signals

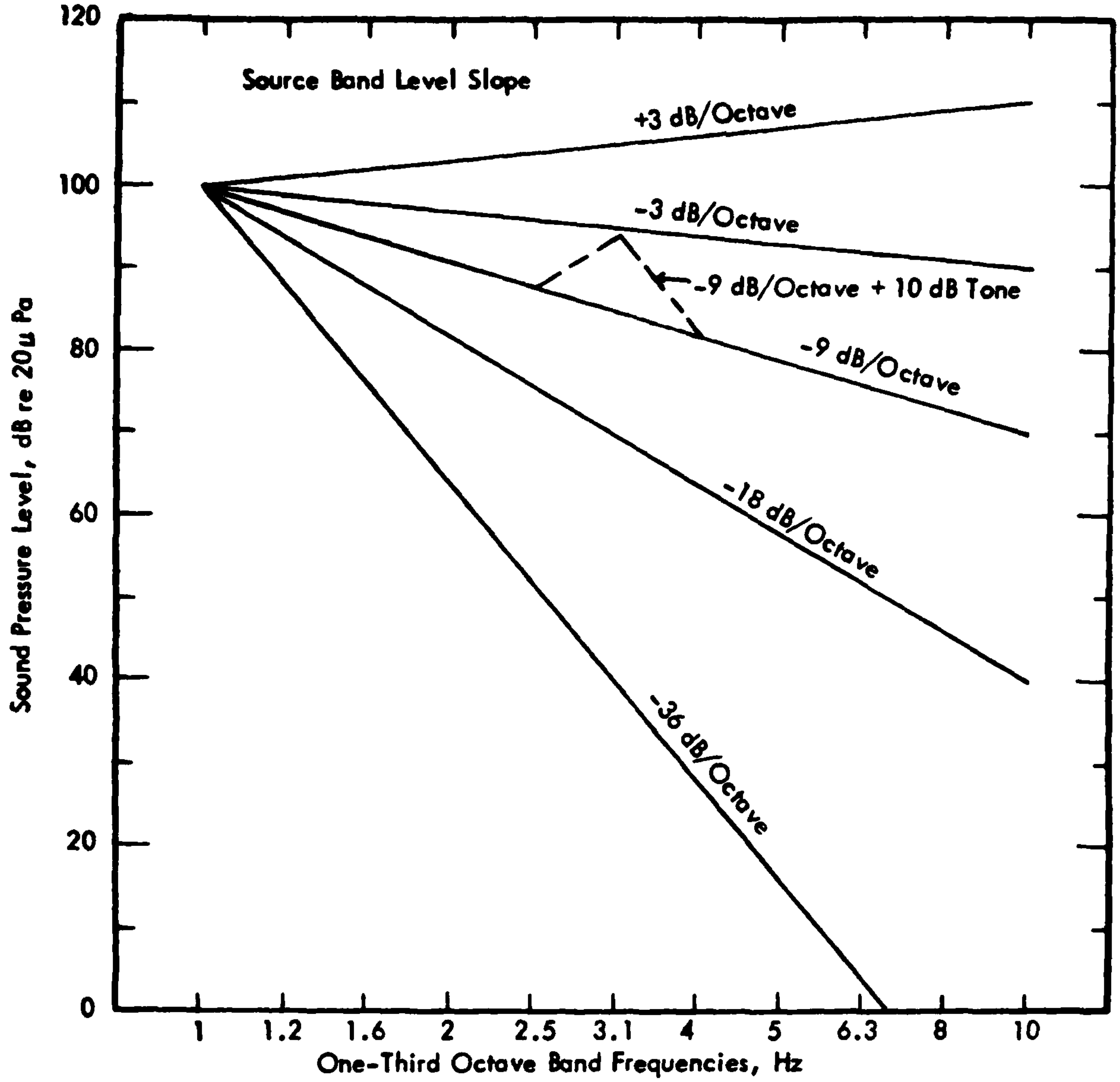


Figure 4. Idealized Source Spectra with Various Band Level Slopes, Including One with a Superimposed 10 dB Pure Tone Component, Used to Evaluate Filter and Spectrum Slope Errors

the receiver assuming either an ideal or real filter (with finite sidebands) and hence compute the filter error $\Delta_F(R)$ at the receiver. Similarly, the received spectrum level can be defined exactly so the slope error $\Delta_S(R)$, at the receiver, can also be computed. In both cases, these new values are computed from the same equations (4a) and (5).

The results of these computations are shown in Figures 5 and 6. Figure 5 shows $\Delta_F(R)$ as a function of frequency for the source spectra of Figure 4 and for two weather conditions. The first is a standard day of 25°C and 70 percent relative humidity, for which atmospheric absorption losses tend to be near a minimum within the FAA Part 36 test window.[5, 16] The second is at 15°C and 35 percent relative humidity - a condition for which atmospheric absorption losses tend to be a maximum within the limits on temperature and humidity allowed for aircraft noise certification.[5, 16]

The general trend in $\Delta_F(R)$, shown in Figure 5, can be explained by three interacting influences.

- Δ_F increases at all frequencies, uniformly, as the source spectrum slope increases, as expected from Figure 3.
- Δ_F increases at high frequencies due to the further increase in signal slope due to atmospheric absorption over the fixed 600 m propagation distance assumed - this effect is clearly dominant in Figure 5b.
- Δ_F is modified, in a complex way, for any bands close to or including a band with a strong tone component.

In Figure 6, using only three of the source spectra evaluated in Figure 5, the variation in $\Delta_S(R)$ at a receiver has also been evaluated. Again, the same general pattern that occurred in Figure 5 is evident. Note that these error terms are not shown beyond an upper bound of 6 dB intentionally. They generally reach such a magnitude only when the total signal attenuation due to atmospheric absorption is very high resulting in a received signal near or below practical limits of measurement in the presence of normal background noise.[2]

One point should be clear by now - the real error term Δ_F and the pseudo error Δ_S behave in a complex manner with the measured signal spectrum

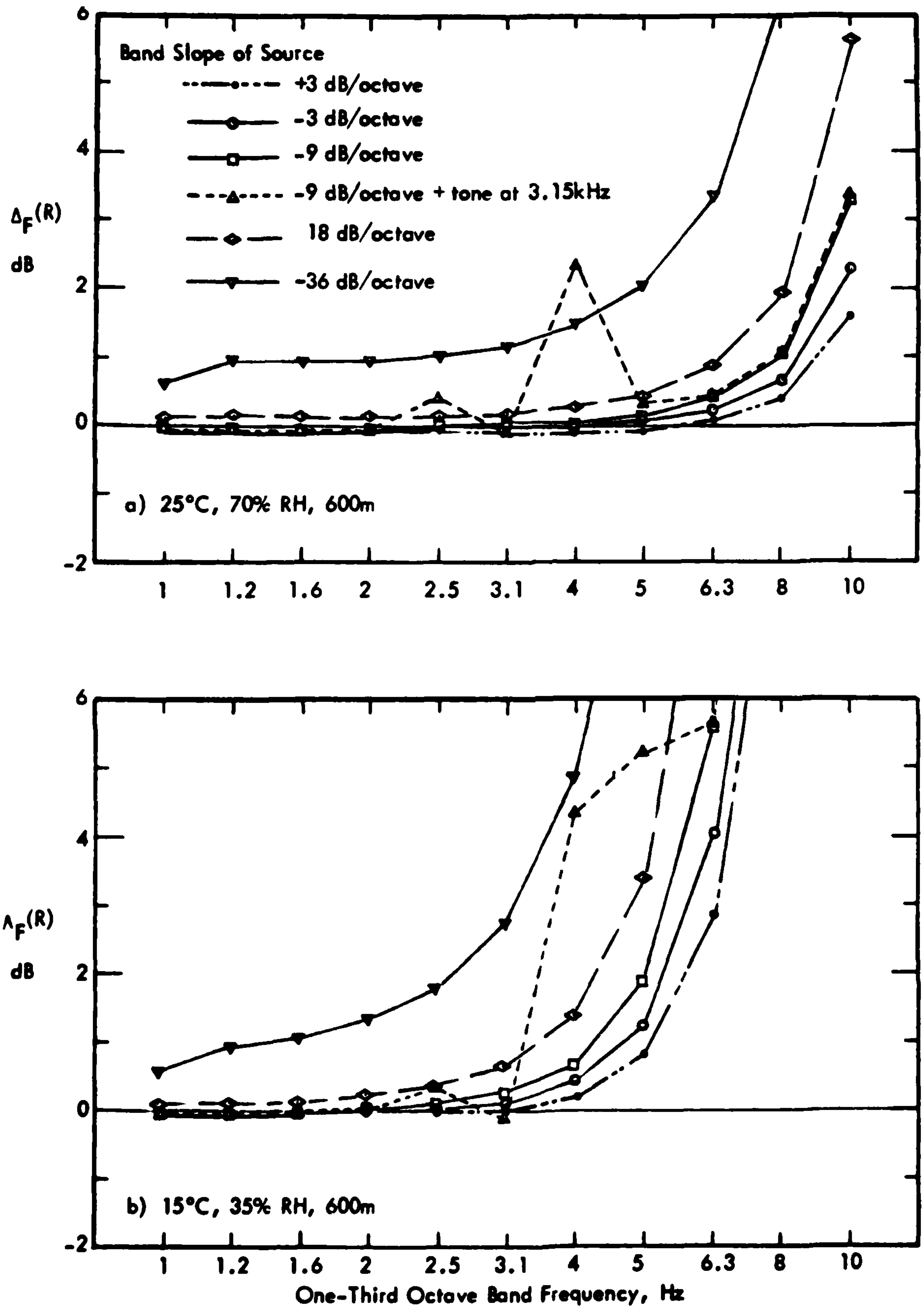


Figure 5. Variation in Filter Error Δ_F at Receiver after Propagation Over 600m with Two Different Weather Conditions, from Source with Various Initial Slopes Shown in Figure

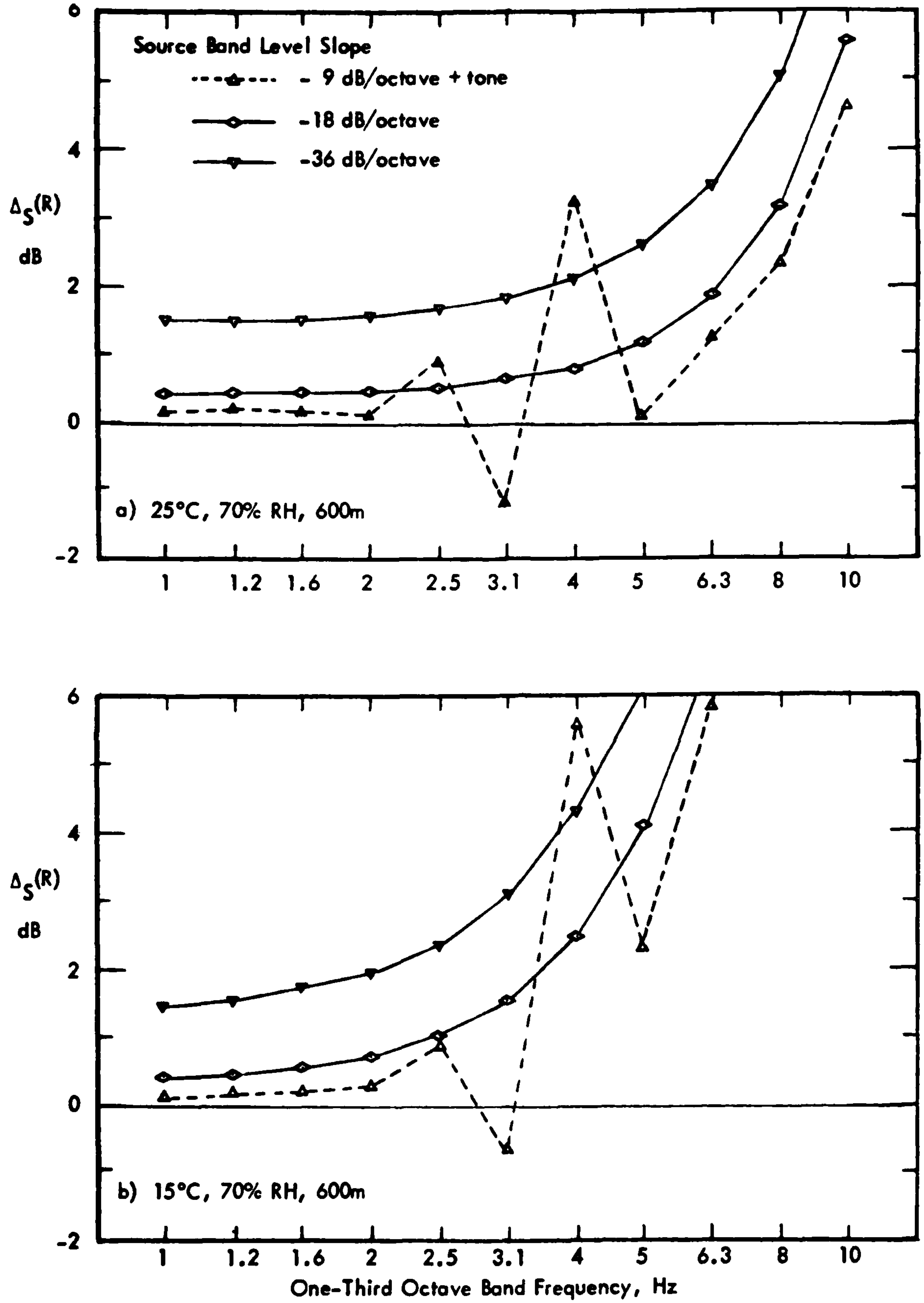


Figure 6. Variation in Slope Error Δ_S at Receiver After Propagation Over 600m, with Two Different Weather Conditions, from Sources with the Various Slopes Indicated

characteristics. This complexity is well-recognized, of course, and is one reason why almost no attempt is made to account for these error terms when analyzing aircraft noise data. One exception is the simple procedure inherent in the aviation industry standard for predicting air absorption[16] which calls for using essentially the lower band edge frequency for evaluating all absorption losses for frequencies equal to or greater than 5000 Hz. This procedure is equivalent to trying to recognize the inherent slope error Δ_S term and compute atmospheric absorption losses at a frequency below the band center which should be more "representative" of changes in the band energy. The procedure is probably in the right direction to aid in minimizing errors when analyzing attenuation of bands of noise. However, a more general approach to the problems of accounting for filter band and signal slope errors seems desirable. Such an approach is developed in the next section.

3. SPECTRUM ITERATION SCHEME

The preceding analysis of the filter and signal slope errors Δ_F and Δ_S was possible because the initial spectrum levels were known. In general, when measuring an aircraft signal at some distance from the source, one really doesn't know the true received spectrum very well in all cases when current techniques of signal analysis are used, i.e., one-third octave band analysis with filters designed to meet applicable industry standards. (See discussion at the beginning of Appendix C for additional details on this point.) While it would be possible to use narrow band analyzers to evaluate aircraft flyover signals and hence determine the true spectrum shape within each one-third octave band, such a process would probably be impractical for universal application by the aviation industry.

Instead of requiring actual narrow band analysis to determine the true signal spectrum, it seemed desirable to develop a method of estimating the apparent true spectrum for any received signal such that by integrating this apparent true spectrum over the full frequency response of the filter, one would be able to replicate the original signal "as measured" with real filters. Once this apparent true spectrum is available, one can then proceed directly to carry out desired frequency-dependent operations on the original signal from the following perspective.

- The apparent true spectrum can be integrated over a simulated ideal filter to obtain the true band level L_{BI} free of filter errors. Alternatively, a standard "real filter" can be synthesized to compute band levels based on accepting a known filter error.
- There is no longer any need to apply corrections for the filter or signal slope errors since these are essentially eliminated once the true apparent spectrum is known.
- The apparent true spectrum can be adjusted for off-reference test conditions of measurement distance or weather using atmospheric absorption models which are strictly accurate only at single frequencies, i.e., at frequencies corresponding to those specified for the spectrum levels.

A spectrum iteration technique to accomplish this is illustrated conceptually in Figure 7 where it is shown being applied to estimate the true source spectrum that would be measured with ideal filters, given levels measured at a receiver with real filters.

This simplified flow chart identifies the essential steps of an algorithm suitable for automation in a computer program, details of which are described in the next section. Essentially, the problem involves estimating the true spectrum levels of the receiver signal and iterating this estimate until, by integration of this spectrum (interpolated at any desired frequency from values estimated at the band center), over the response bandwidth of the simulated measurement filter, the "as measured" levels can be duplicated, within a definable error criteria. Details of this process are covered in the next section.

3.1 Spectrum Iteration Procedures

Given the band level $L_B(f_i)$ measured, with a practical filter, at each i^{th} frequency f_i in the range f_L to f_U we wish to find the true spectrum level function $L_s(f)$ at any frequency which, when integrated over the effective transmission response range of each mathematically simulated filter, will yield the same band level as actually measured. This actually reduces to determining a spectrum level $L_s(f_i)$ at the (geometric) center frequency f_i of each band. These spectrum levels, combined with a suitable interpolation method, are then used to estimate spectrum levels at other frequencies and hence compute band levels based on numerical integration using a mathematical representation of either an ideal or practical filter.

It should be recognized, of course, that such a procedure for estimating the spectrum levels at all frequencies within a given band is subject to some inherent limitations.

- The estimated spectrum levels can be uniquely defined for each band in a given spectrum with the use of any one suitable iteration and interpolation scheme. However, any change in either the iteration or interpolation scheme may produce small variations in the estimated spectrum levels.

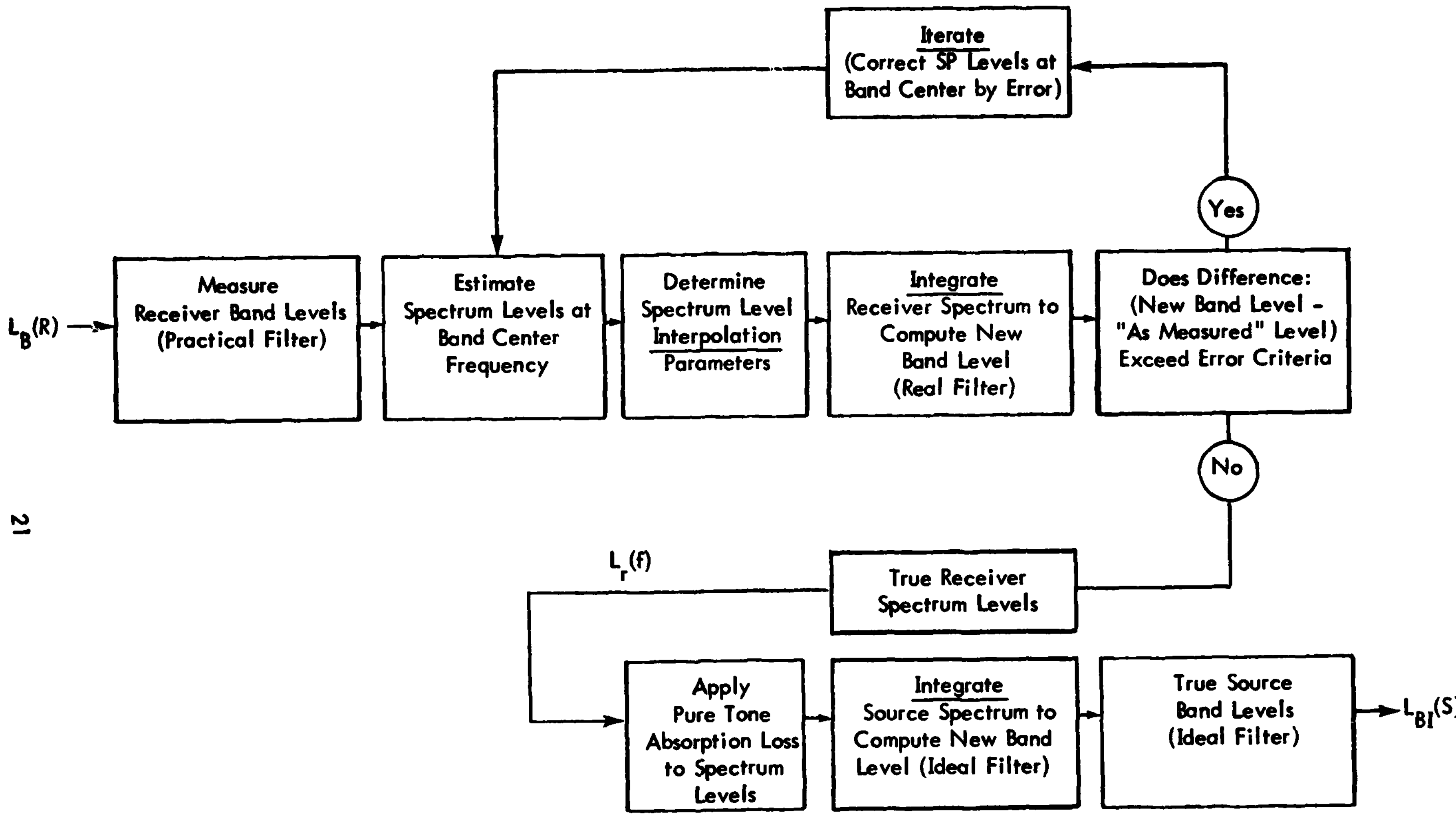


Figure 7. Spectrum Iteration Process to Estimate True Receiver Spectrum from Measured Receiver Spectrum and thus Allow Reconstruction of Source Band Levels Free of Errors Due to Finite Filter Slopes (Δ_F) or Spectrum Slope (Δ_S)

- o The estimated spectrum levels are, of course, a smoothed replica of the true spectrum. For example, they represent the presence of a pure tone component superimposed on a broad band noise by means of a spectral peak with a finite bandwidth centered near the actual band center frequency. (An example of such a representation will be shown later.)

Nevertheless, the iteration and interpolation scheme presented in this report has been found to provide meaningful results and has proven itself to be relatively insensitive to minor variations in the iteration procedure, thus minimizing the uncertainty in the "uniqueness" of the estimated spectrum. The basic steps in this procedure can be defined as follows.

3.1.1 Initial Estimate of Spectrum Level

For a zero order approximation to the spectrum level at each band-center frequency, assume that the noise has a flat spectrum (i.e., constant spectrum level over the filter band), and the actual measurement filter is in fact ideal (i.e., its transmission is unity at all frequencies within its nominal passband and zero at all other frequencies). Thus, this zero-order estimate of the apparent spectrum level $L_s(f_i)$, or simply L_{so}, at the band center frequency will be (dropping the f_i index for simplicity)

$$L_{so} \approx L_B - 10 \log_{10} (\Delta f) \tag{7}$$

where[9]

$$\Delta f = \begin{cases} \text{the nominal filter bandwidth} \\ \left[2^a - 2^{-a} \right] f_i \end{cases}$$

$$a = \begin{cases} 1/2 \text{ for full octave band filters} \\ 1/6 \text{ for one-third octave band filters} \end{cases}$$

$$f_i = \begin{cases} \text{nominal band center frequency} \\ 10^{3n/10} \text{ for full octave band filters} \\ 10^{n/10} \text{ for one-third octave band filters} \end{cases}$$

n = any integer

3.1.2 Refined Estimate of Spectrum Levels

Assume three spectrum levels or, for convenience in notation at this point, their equivalent spectral densities, S_{i-1}, S_i, and S_{i+1} at frequencies f_{i-1}, f_i, and f_{i+1}, respectively. Further, assume the approximate value of the band level is the area under the two trapezoids shown in Figure 8.

$$S_{i-} \longrightarrow S_i \text{ between } f_{i-} \text{ and } f_i$$

and

$$S_{i+} \longrightarrow S_i \text{ between } f_{i+} \text{ and } f_i$$

where f_{i+} and f_{i-} are the upper and lower band edges of a constant percentage band filter centered at f_i.

Further assume that along this curve of spectral density vs frequency, the spectral density slope changes linearly with the relative frequency so that at any frequency f, the spectral density is:

$$S(f) = S_i \left(\frac{f}{f_i}\right)^{a_i + b_i(f/f_i)} \tag{8}$$

Letting $f = f_{i-1}$, then between f_{i-1} and f_i, the ratio of the spectral densities is

$$\frac{S(f_i)}{S(f_{i-1})} = \left(\frac{f_i}{f_{i-1}}\right)^{a_i + b_i(f_{i-1}/f_i)} \tag{9}$$

or, in decibels,

$$\delta_{i-} = 10 \log \frac{S(f_i)}{S(f_{i-1})} = 10 \left[a_i + b_i\left(\frac{f_{i-1}}{f_i}\right)\right] \log \frac{f_i}{f_{i-1}} \tag{10}$$

Similarly, between f_i and f_{i+1}, the ratio is

$$\frac{S(f_{i+1})}{S(f_i)} = \left(\frac{f_{i+1}}{f_i}\right)^{a_i + b_i(f_{i+1}/f_i)}$$

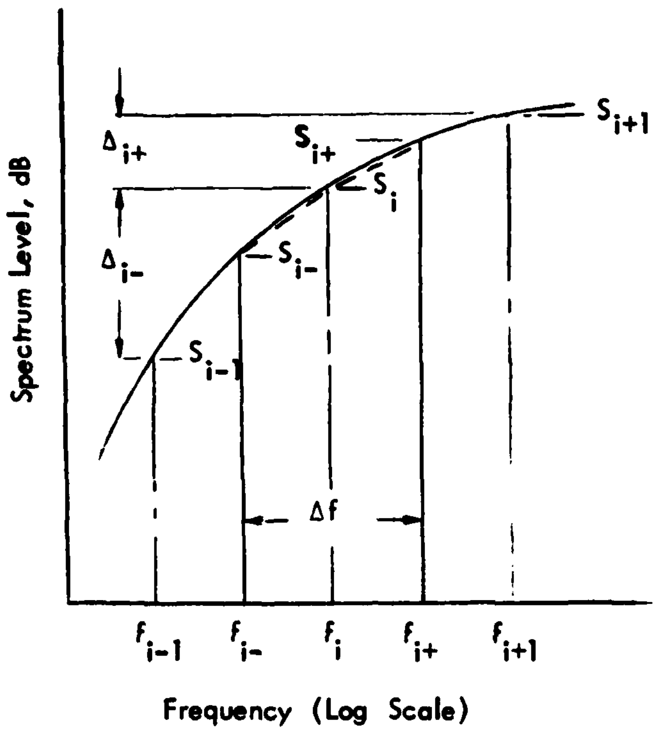


Figure 8. Illustration of Initial Two-Element Integration Model for First Refinement in Spectrum Level Estimate

or, in decibels

$$\delta_{i+} = 10 \log \frac{S(f_{i+1})}{S(f_i)} = 10 \left[a_i + b_i \left(\frac{f_{i+1}}{f_i} \right) \right] \log \frac{f_{i+1}}{f_i} \quad . \tag{11}$$

Since, for constant percentage bands, there is a constant ratio r given by

$$\frac{f_{i+1}}{f_i} = \frac{f_i}{f_{i-1}} = r$$

equations (10) and (11) become

$$\delta_{i-} = 10 \left[a_i + b_i/r \right] \log r \tag{12}$$

$$\delta_{i+} = 10 \left[a_i + b_i r \right] \log r \tag{13}$$

Solving for a_i and b_i, these interpolation parameters for the i-th band are

$$a_i = \frac{-1}{(r^2 - 1)} \frac{1}{10 \log r} \left[\delta_{i+} - r^2 \delta_{i-} \right] \tag{14}$$

$$b_i = \frac{r}{(r^2 - 1)} \frac{1}{10 \log r} \left[\delta_{i+1} - \delta_{i-1} \right] \tag{15}$$

Applying these constants to the general expression at any frequency, the spectral density S(f), or the equivalent spectrum level, $L_S(f) = 10 \log \left[S(f)/P_o^2 \right]$ can be estimated at any frequency using eq. (8).

This process is used to estimate the spectrum levels at the filter band edges f_{i-} and f_{i+} and, as indicated in Figure 8, the band level, assuming a perfect filter, is assumed to be given by the area under the two trapezoids. (The integration equations for the power in each trapezoid have already been cited in Appendices B and C.) Essentially, this makes it possible to compute the slope error Δ_S for this spectrum so that a refined estimate of spectrum level $L_S(f_i)$ at the center frequency of the i-th band can be obtained, using eq. (4b). But this refined

estimate now allows a second estimate of the interpolation parameters a_i and b_i to be made from equations (14) and (15) using new values for the spectrum level differences δ_{i+} and δ_{i-} between adjacent bands. This refinement process is repeated twice before proceeding with the next step.

3.1.3 Integration of Band Levels

Now, applying the same band integration equations cited earlier, the total power passed by a simulation of the real filter is computed numerically, by dividing each band into b constant percentage elements, as defined in detail in Appendix C. A value of b = 6 will be shown to be adequate.

This new value of band level is then compared to the original "as measured" value and the error compared to an error criteria. (A fixed value of 0.1 dB was used in the iteration algorithm applied here.) This error test is applied to each band, one at a time, over the entire frequency range and, if it is exceeded for any one band, the entire set of new spectrum levels are corrected by the amount of error just determined and the entire process repeated, as indicated in Figure 7.

Most of the time, two or three iterations were required for real aircraft spectra to achieve stable results. The outcome of this process is then a set of spectrum levels at each of the integration elements which describe, in a self-consistent way, the actual input signal.

One minor variation was allowed in this process for some cases. If the average slope over a given band was less than a predetermined slope criteria, the band integration step was omitted since it was possible to compute the band level directly (i.e., Δ_F was expected to be sufficiently small to eliminate the need for integration.)

One other feature explored in the iteration process was to employ an error multiplier when correcting the old spectrum levels by the error in band levels. That is, the new spectrum levels at the band center frequency were defined by

$$L_S(f_c)_{new} = L_S(f_c)_{old} - C \quad (L_{B\,new} - L_{B\,old})$$

where C = the error multiplier. A value for C of 1 to 1.2 was found to be satisfactory.

3.2 Evaluation of Spectrum Iteration

A test spectrum, shown in Figure 9a, was constructed from an actual aircraft flyover signal modified to contain an apparent tone component of 3150 Hz. Figure 9b shows the range of spectrum levels, relative to final base case values, for a variety of minor changes to the iteration algorithm. The maximum range was about ± 0.1 dB - consistent with the error criteria of 0.1 dB for the iteration process.

Also shown in Figure 9a is the absolute value of the zero-order estimate of spectrum level for each band, as given by eq. (7); Figure 9b shows the deviation of this estimate from the base case. Note that the relative error in this value, indicated in Figure 9b by the diamonds, is much greater than the range of values of the iterated levels, indicated by the shaded areas.

In Table I, a more detailed summary is shown of results of the minor variations in the iteration algorithm. The adjustable parameters were:

- o The number of bands integrated on each side of the central i-th band (2 was finally selected, thus representing each filter's response range by a total of 5 bandwidths).
- o The number of elemental subdivisions for integration of each band (a value of 6 was found to be sufficient for good accuracy).
- o The error multiplier factor. (A value of 1 was finally selected.)
- o The slope criteria used to short-cut the integration of some bands. (This was usually chosen to be either 0 or 1 dB, and caused either all or 19 out of the 24 bands in the test spectrum in Figure 9a to be integrated during iteration.)

The results of these various cases show that:

- o Three iterations were usually required,
- o The rms error in the iterated band levels (re the "as measured" values) was always less than 0.02 dB,
- o Spectrum levels at specific frequencies did not vary significantly in most cases.

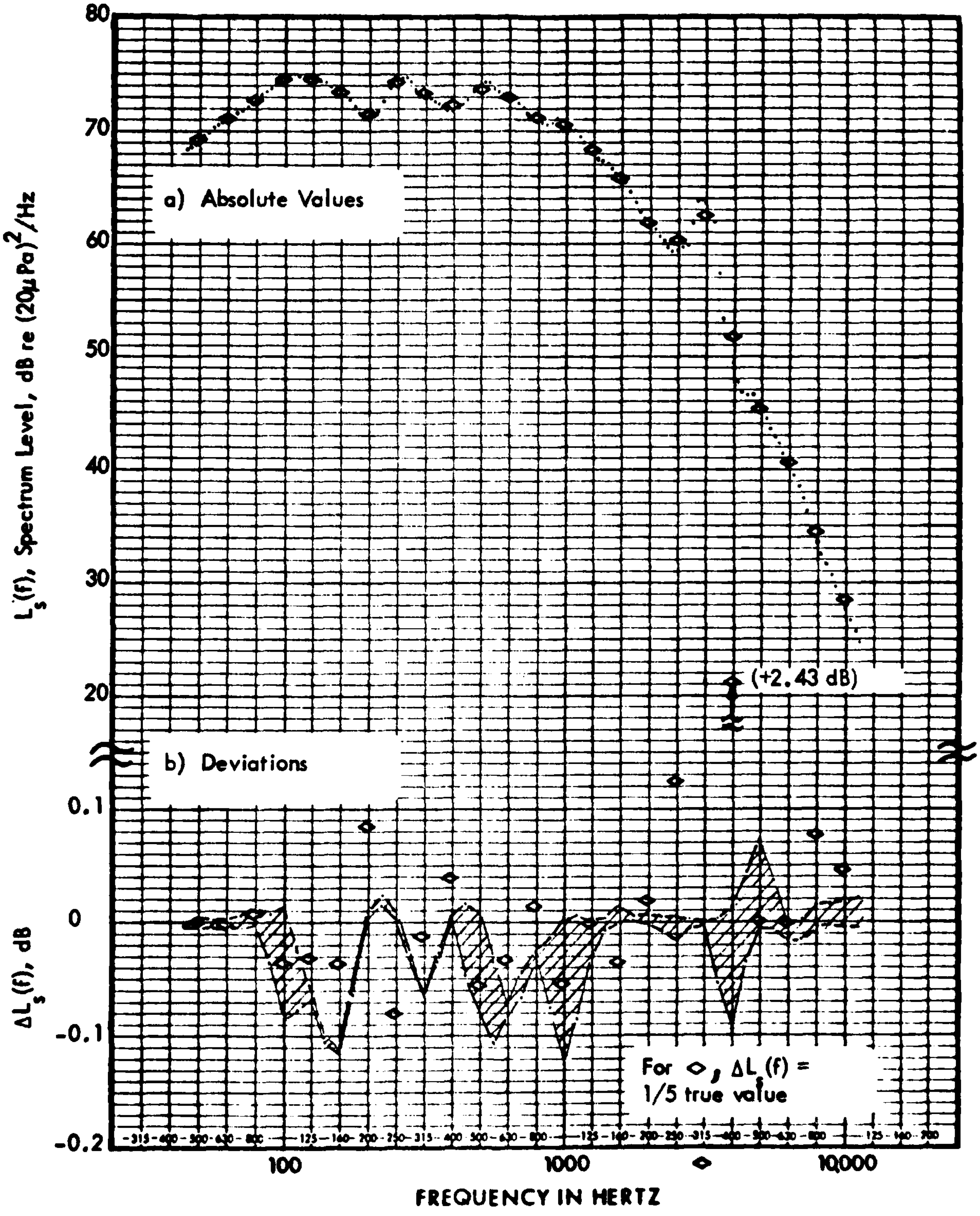


Figure 9. Spectrum Levels Derived from Test Case. Part a) shows iterated values (....) and predicted value (◇) assuming white noise approximation; Part b) shows range (▨.▨) of $L_s(f)$, relative to base case values, for variations in iteration algorithm and the deviation (◇) for the ideal case, shown in Part a).

Table 1

Summary of Spectrum Iteration Results with Test Spectrum for Various Iteration Algorithm Parameters

Adjustable Parameters					Results				
Case	No. of[1] Bands Summed	No. of[2] Elements per Band	Error[3] Factor	Slope[4] Criteria dB/Oct.	No. of[5] Iterations	RMS[6] Error dB	Relative Spectrum Level,[7] dB 100 Hz	1000 Hz	4000 Hz
A	1	6	1	1	3	0.0092	0.001	0.002	0.013
B	2	4		1	3	0.0148	0.465	-0.094	-0.017
C		6		0	3	0.0093	-0.011	-0.003	0
D				1	3	0.0093	0	0	0
E			1.2	1	3	0.0066	-0.001	-0.003	-0.094
F			1	2	3	0.0093		N/A	
G			1.2	2	3	0.0065	-0.096	-0.126	-0.094
H			1.5	2	2	0.0337		N/A	
I		8	1	1	3	0.0087		N/A	
J	3	6	1	1	3	0.0094	<0.001	<0.001	<0.001

1 No. of bands below passband included in integration for filter response. (Only one band above passband was included in all cases.)

2 No. of integration elements into which each band is divided.

3 Error in Nth band estimate multiplied by this factor to correct old spectrum level.

4 Criteria for omitting bands from integration due to expected low filter error Λ_F.

5 No. of times iteration algorithm loop is exercised to achieve required accuracy.

6 RMS value of difference between calculated and actual "as measured" band levels over all 24 bands.

7 Spectrum level at three frequencies from final iterated spectrum taken relative to Case D - the base case used for remainder of study.

As a final test of the concept, the apparent spectrum levels developed were used to compute new "measured" band levels, simulating a real filter in each case, at a different distance and/or weather condition. These new band levels were then reinserted back into the spectrum iteration algorithm and used to compute their own spectrum levels and thus allow correcting these new "as measured" levels back to the original band levels.

Table 2 summarizes the results of this exercise showing in:

Col. 1	The original band levels
Col. 2	The error in duplicating these by iteration
Cols. 3, 4	The new computed band levels at new conditions
Cols. 5, 7	The error in duplicating each of these new band levels when each was inserted, in turn, back into the iteration algorithm
Cols. 6, 8	The final resulting "return error" or difference between the original and recomputed "as measured" levels

Except for the last two bands, the agreement is excellent. This residual error is attributed to:

1. Lack of precise knowledge of the true weather conditions upon which the original test data were based, and
2. Difficulties in extrapolating the highest frequencies due to high spectrum slopes.

Nevertheless, the overall results are considered quite satisfactory, indicating the scheme should be more than sufficient to test for errors, due to filter sidebands, in PNL or EPNL values of real aircraft spectra. This was carried out in the next section.

Table 2

Evaluation of Iteration Algorithm Starting with "Test" Data File, Generating New Spectra at Different Distance and Weather and Using These New Files to Recompute the Original Data File

Data File	Test				T600		T750	
Column	1	2	3	4	5	6	7	8
Temp. °C	24°		25	15	25	24	15	24
Humidity %	64		70	35	70	64	35	64
Distance m	800		600	750	600	800	750	800
Frequency Hz	L_B	$\epsilon(f)$	L_B	L_B	$\epsilon(f)$	$\epsilon(f)$	$\epsilon(f)$	(f)
	dB							
50	79.9	0.001	79.91	79.85	0.01	0.01	0	0
63	82.7	0.001	82.72	82.63	0.01	0.01	0	0
80	85.5	0.001	85.53	85.40	0	0	0	0
100	88.2	0.001	88.17	87.98	-0.08	-0.15	0	-0.16
125	89.1	0.001	89.10	88.85	-0.07	-0.14	0	-0.14
160	88.9	0.001	88.91	88.59	-0.10	-0.20	0	-0.20
200	88.3	-0.002	88.47	88.09	0	0	0	-0.01
250	91.9	0.002	92.15	91.76	0.03	0.02	0	0
315	91.8	0.001	92.11	91.76	-0.05	-0.10	0	-0.10
400	91.8	-0.002	92.29	92.08	-0.02	-0.03	0	-0.03
500	94.4	0.001	94.95	94.95	-0.10	-0.21	0	-0.20
630	94.6	0.001	95.33	95.55	-0.08	-0.15	0	-0.15
800	93.7	-0.001	94.60	94.99	-0.04	-0.08	0	-0.08
1,000	94.1	0.001	95.06	95.39	-0.10	-0.21	0	-0.21
1,250	92.7	-0.001	93.88	93.74	-0.05	-0.08	0	-0.07
1,600	91.3	0.001	92.72	91.49	-0.09	-0.09	0	0
2,000	88.5	0.001	90.27	86.98	-0.07	-0.06	0	0
2,500	87.7	0.002	90.14	82.73	0.01	0	0	-0.01
3,150	91.1	0.012	94.37	81.86	0.04	0.05	0	0.01
4,000	81.5	0.041	85.82	67.75	0.07	0.06	0.01	-0.02
6,300	72.2	0.012	83.06	34.01	-0.11	-0.11	0	0.07
8,000	66.9	-0.009	83.72	14.51	-0.01	-0.12	0.32	0.88
10,000	61.8	0.001	88.73	-6.41	0	0.49	0.61	4.46
RMS Error, dB		0.006			0.07	0.15	0.14	0.93
No. of Iterations		3			2		6	

Col. 1 - Hypothetical "As Measured" Band Levels for "Test" Data - Real Filter

2 - Difference Between Band Levels, Computed from Iterated Spectrum, and Col. 1.

3, 4 - Band Levels Computed from Iterated Spectrum at New Distance and Weather - Real Filter.

5 - Difference Between New Band Levels, Computed from Iterated Spectrum, Based on Treating Col. 3 as "As Measured" Levels and Col. 3.

6 - Difference Between Band Levels Computed for Original Weather Conditions of Col. 1 and Original "As Measured" Levels in this column

7, 8 - Repeat of Cols. 5 and 6 for Different Distance and Weather.

4. EVALUATION OF FILTER EFFECTS ON PNL OR EPNL VALUES

4.1 Filter Errors in PNL Values

Four representative aircraft spectra recorded at PNLTM using the data collection and correction procedures described in Volume I[1] were selected for evaluation of any PNL errors. (The actual data were corrected very slightly to normalized values for a standard day and a measurement distance of 300 m. This adjustment was generally less than 2 dB.) These are shown in Figure 10. Figure 11 shows the high frequency portion of the iterated spectrum level for one case (Data File 47 707 Approach) compared to the zero order estimate of spectrum level using eq. (4b). The latter appears to be a good approximation of most frequencies except near the strong tone component where, for example, at 3150 Hz, this "white noise" approximation would be about 5 to 6 dB higher than the average iterated value in this band. This is one clear example of how the zero order estimate of spectrum level can be quite misleading. (The filter error Δ_F at this frequency was found to be 3.4 dB.)

In Table 3, a detailed listing is provided of the "as measured" band levels, L_B, the iteration error ϵ for each band at the original "as measured" distance of 300 m, and the computed filter error Δ_F based on use of the iterated spectrum in equations 2, 3 and 5.

In addition, the values of new computed band levels L_B' at 600 and 900 m are given along with the corresponding new filter error Δ_F and the difference Δ_{SAE} between this new band level and a value that would have been computed if only the single band attenuation factors specified in SAE ARP 866A had been employed. (For the sake of brevity, only the higher frequency values are tabulated for the last three cases.)

For each of the "as measured" and computed band levels, PNL values were computed using the procedures specified in FAR Part 36, Appendix B, augmented, where necessary, by Reference 21.

These PNL values are summarized in Table 4 for each data file covering more distances than shown in Table 3. For each data file, the computed PNL is listed based on computations with the iterated spectrum (whose rms error is also

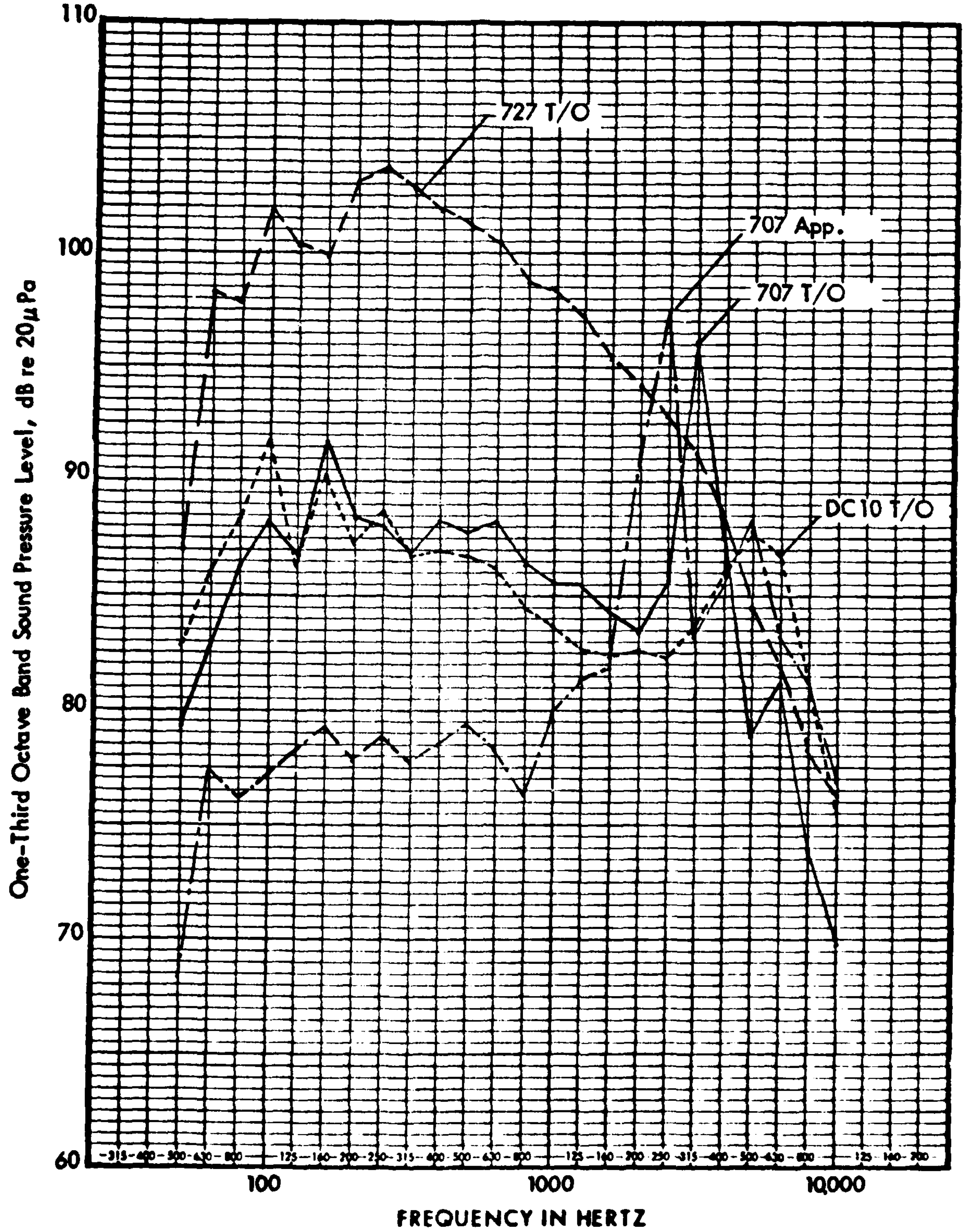


Figure 10. Normalized PNLTM Spectra of Four Representative Aircraft Flyby Signals Used to Demonstrate Application of Spectrum Iteration to Predict Band Levels Free of Filter Effects - 300m, 25°C, 70% RH.

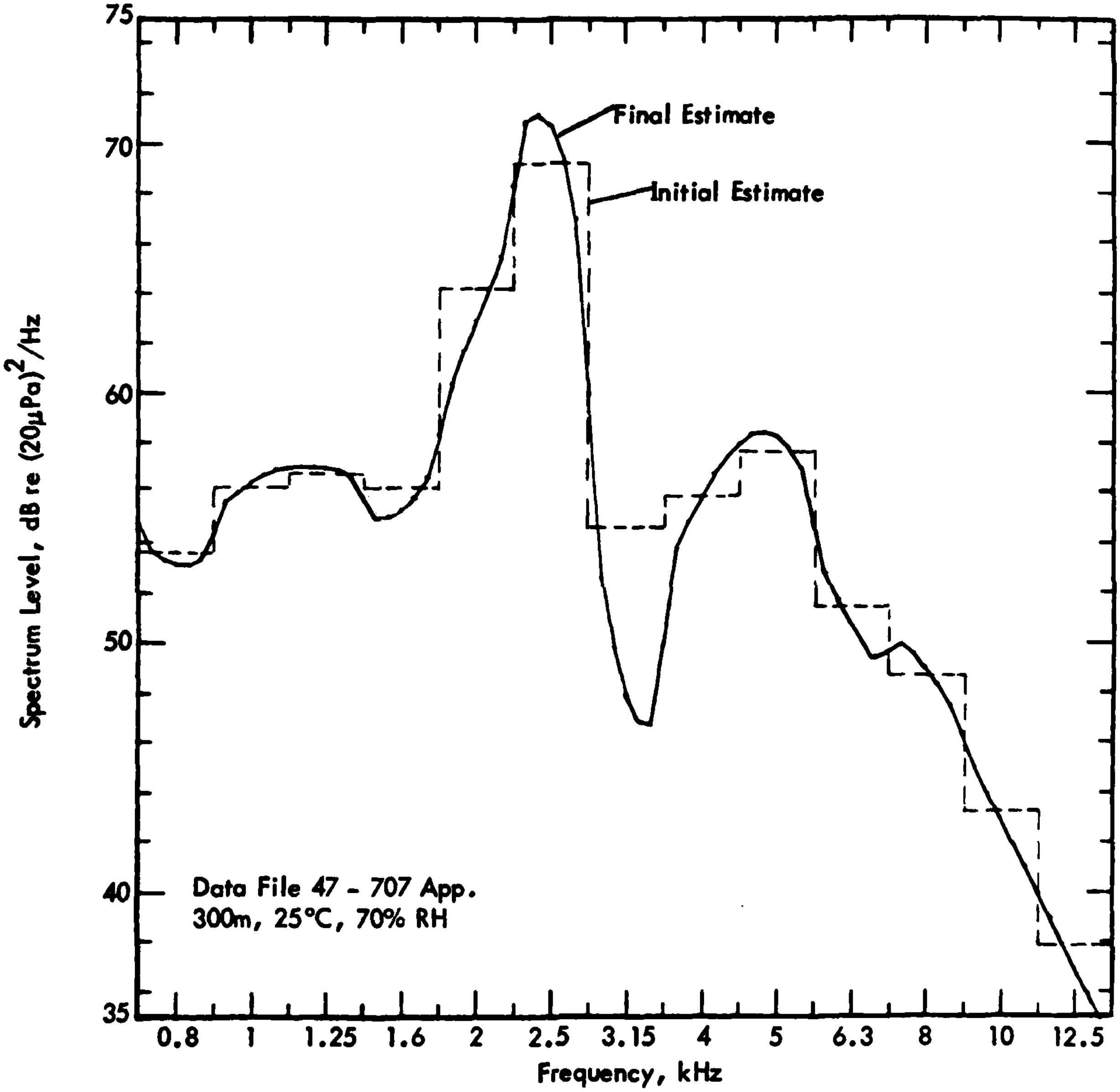


Figure 11. Comparison of Initial Estimate of Spectrum Level, Based on White Noise Approximation, Signified by Dashed Line and Final Estimate, Signified by Solid Line, developed from iteration process so that this spectrum, integrated over full response range of a real filter, produced the "as measured" band levels.

Table 3

Iteration Errors and Calculated Filter Errors (Δ_F) and Error in Use of Simulated SAE ARP 866A Procedure for Four Representative Aircraft Noise Spectra Normalized Initially to 300 m, 25°C, 70 Percent RH

(a) Data File 35 - 727 T/O

Freq.	Distance, 300 m			Distance, 600 m			Distance, 900 m		
	$L_B^{(1)}$	$\epsilon(f)^{(2)}$	$\Delta_F^{(3)}$	$L'^{(4)}_B$	$\Delta_F^{(3)}$	$\Delta_{SAE}^{(5)}$	$L'^{(4)}_B$	$\Delta_F^{(3)}$	$\Delta_{SAE}^{(5)}$
Hz		dB			dB			dB	
50	87.0	-0.02	0.98	86.97	-0.02	-0.02	86.95	-0.03	-0.03
63	98.2	0.01	-0.20	98.19	-0.20	0.01	98.17	-0.20	0.01
80	97.9	-0.01	0.16	97.86	0.14	-0.01	97.83	0.16	-0.01
100	101.9	0	-0.17	101.85	-0.17	0	101.81	-0.17	0
125	100.4	0	-0.02	100.32	-0.03	-0.01	100.25	-0.03	-0.01
160	99.9	0	0.03	99.78	0.03	-0.01	99.66	0.02	-0.01
200	103.1	0	-0.12	102.92	-0.12	-0.01	102.74	-0.12	-0.01
250	103.7	0	-0.10	103.43	-0.10	0	103.16	-0.10	0
315	102.7	0	-0.06	102.29	-0.07	0	101.89	-0.07	0
400	101.9	0	-0.08	101.30	-0.08	0	100.71	-0.07	0.01
500	101.2	0	-0.07	100.34	-0.08	-0.01	99.49	-0.07	0
630	100.5	0	-0.10	99.33	-0.10	0.01	87.17	-0.09	0.02
800	98.7	0	-0.03	97.16	-0.03	0.01	95.62	-0.03	0.02
1,000	98.4	0	-0.10	96.45	-0.09	0.01	94.50	-0.09	0.02
1,250	97.2	0	-0.08	94.82	-0.08	0.01	92.45	-0.07	0.04
1,600	95.4	0	-0.05	92.55	-0.05	0.02	89.71	-0.05	0.04
2,000	94.1	0	-0.07	90.69	-0.07	0.01	87.30	-0.06	0.04
2,500	92.8	0	-0.07	88.67	-0.07	0.02	84.57	-0.05	0.06
3,150	91.2	0	-0.08	86.08	-0.06	0.06	80.99	-0.04	0.15
4,000	88.4	0	-0.06	81.83	-0.01	0.17	75.34	0.05	0.42
5,000	84.5	0	0.01	75.65	0.10	-1.04	67.02	0.25	-1.76
6,300	82.0	0	-0.08	69.56	0.04	-1.62	57.61	0.31	-2.74
8,000	78.3	-0.01	0	60.55	0.30	-2.20	44.13	1.26	-3.07
10,000	76.3	0	-0.07	49.94	0.47	-3.37	27.23	3.09	-3.09
PNL	118.61	0	-0.07	115.91	-0.07	-0.07	114.80	-0.05	-0.03

Table 3 (Continued)

(b) Data File 47 - 707 APP

Freq. Hz	Distance, 300 m			Distance, 120 m			Distance, 600 m		
	$L_B^{(1)}$	$\epsilon(f)^{(2)}$	$\Delta_F^{(3)}$	$L_B'^{(4)}$	$\Delta_F^{(3)}$	$\Delta_{SAE}^{(5)}$	$L_B'^{(4)}$	$\Delta_F^{(3)}$	$\Delta_{SAE}^{(5)}$
	dB			dB			dB		
1,000	79.9	0	-0.09	81.09	-0.01	0.01	77.91	-0.10	-0.03
1,250	81.4	0	-0.12	82.84	-0.11	0	79.00	-0.12	-0.01
1,600	81.9	0	0.29	83.67	0.32	-0.05	78.96	0.26	-0.07
2,000	90.9	0	0.36	93.06	0.39	0.11	87.31	0.31	-0.17
2,500	96.8	0.01	-0.20	99.27	-0.19	-0.02	92.72	-0.20	0.07
3,150	83.2	0.10	3.38	86.10	3.21	-0.21	78.68	3.69	0.66
4,000	85.5	0.01	-0.03	89.77	0.01	0.23	78.48	-0.07	-0.28
5,000	88.2	0	-0.18	93.71	-0.17	0.82	79.17	-0.16	-1.22
6,300	83.1	0	0.17	90.71	0.13	1.12	70.93	0.43	-1.35
8,000	81.3	0	-0.12	92.68	-0.11	2.05	63.30	0.08	-2.40
10,000	76.9	0	0.03	93.69	0	3.00	51.47	0.87	-2.40
PNL	114.37	0.01	0	117.81	0	0.32	109.85	0.03	-0.08

(c) Data File 88 - 707 T/O

Freq. Hz	Distance, 300 m			Distance, 600 m			Distance, 900 m		
1,000	85.5	0.08	-0.07	83.47	-0.06	-0.07	81.51	-0.07	-0.07
1,250	85.5	0.10	-0.11	83.02	-0.10	-0.09	80.63	-0.10	-0.08
1,600	84.3	0.06	-0.07	81.39	-0.06	-0.04	78.55	-0.05	-0.02
2,000	83.4	0.15	-0.08	79.84	-0.08	-0.15	76.44	-0.08	-0.12
2,500	85.6	-0.01	1.32	81.14	1.14	-0.31	76.71	0.99	-0.60
3,150	96.0	-0.02	-0.20	90.79	-0.20	-0.03	85.58	-0.20	-0.06
4,000	87.5	0	0.91	81.38	1.14	0.62	75.34	1.40	1.32
5,000	78.9	0.02	0.39	70.06	0.60	-1.03	61.68	1.08	-1.60
6,300	81.1	0	-0.20	68.45	-0.16	-1.83	56.15	-0.08	-3.30
8,000	73.9	0.04	0.40	57.05	1.16	-1.30	41.42	2.47	-1.38
10,000	69.8	-0.01	-0.05	44.00	0.87	-2.81	23.30	5.44	-0.5
PNL	115.29	0.01	-0.03	110.86	-0.03	-0.09	107.07	-0.02	-0.08

Table 3 (Concluded)

(d) Data File 90 - DC-10 T/O

Freq. Hz	Distance, 300 m $L_B^{(1)}$ dB	$\epsilon(f)^{(2)}$	$\Delta_F^{(3)}$	Distance, 600 m $L'_B{}^{(4)}$ dB	$\Delta_F^{(3)}$	$\Delta_{SAE}^{(5)}$	Distance, 900 m $L'_B{}^{(4)}$ dB	$\Delta_F^{(3)}$	$\Delta_{SAE}^{(5)}$
1,000	83.7	-0.09	-0.08	81.66	-0.08	-0.08	79.71	-0.08	-0.07
1,250	82.7	-0.07	-0.06	80.25	-0.06	-0.06	77.87	-0.05	-0.04
1,600	82.7	-0.08	-0.07	79.46	-0.07	-0.07	76.60	-0.07	-0.07
2,000	82.6	-0.10	-0.09	79.18	-0.09	-0.10	75.66	-0.10	-0.10
2,500	82.2	-0.05	-0.04	77.98	-0.05	-0.07	73.84	-0.04	-0.07
3,150	83.7	-0.07	-0.06	78.37	-0.08	-0.15	73.16	-0.08	-0.18
4,000	86.0	0	-0.07	79.15	-0.10	-0.11	72.40	-0.10	-0.12
5,000	87.8	-0.07	-0.08	78.58	-0.11	-1.41	69.62	-0.07	-2.56
6,300	86.7	0.01	-0.11	74.27	-0.02	-1.61	62.25	0.17	-2.80
8,000	81.4	-0.01	0.07	64.13	0.55	-1.73	48.06	1.59	-2.24
10,000	75.5	-0.02	0.05	50.45	1.21	-2.06	29.73	5.43	0.21
PNL	*110.85*	*-0.05*	*-0.08*	*105.33*	*-0.05*	*-0.25*	101.95	-0.05	-0.22

Notes

(1) L_B - Actual "as measured" band level fed into spectrum iteration routine.

(2) $\epsilon(f)$ - Difference between band level L_B' computed from iterated spectrum at original distance of 300m and the "as measured" band levels at this distance.

(3) Δ_F - Filter error based on calculated "as measured" band levels with real filter.

(4) L_B' - Band levels calculated from iterated spectrum.

(5) Δ_{SAE} - L_B' minus value computed from calculated "as measured" band level using only simulation of SAE ARP 866A procedure.

Table 4

Summary of PNL Values for Four Representative Aircraft PNLTM Spectra
PNL for Ideal Filter and SAE Method Shown Relative
to Calculated "As Measured" Values for a Real Filter

Data File		Temp. Humid.	Filter Type	RMS[2] Error	PNL in dB at Specified Distance in Meters						
					300[3] Meas'd	75	300	450	600	750	900
						Calculated					
35	727 T/O	25° 75%	Real Ideal	0.005 (3)	118.61	122.39 -0.04	118.61 0.07	117.16 0.21	115.91 0.07	115.60 0.06	114.80 0.05
47	707 APP		Real Ideal (SAE)[1]	0.020 (4)	114.37	118.82 -1.03 -0.48	114.38 0 -0.01	112.00 -0.01 0.06	109.85 -0.03 0.08	N/A - -	N/A - -
88	707 T/O		Real Ideal (SAE)	0.011 (3)	115.29	119.33 -0.051 -0.31	115.28 0.03 0.01	112.98 0.03 0.07	110.86 0.03 0.09	108.91 0.02 0.09	107.07 0.02 0.08
90	DC-10 T/O		Real Ideal (SAE)	0.017 (2)	110.85	116.98 0.02 -1.14	110.80 0.08 0.05	107.77 0.06 0.25	105.33 0.05 0.25	103.20 0.05 0.25	101.95 0.05 0.22

(1) SAE denotes use of simulated SAE ARP 866A procedure only to compute band levels at other distance or weather conditions.

(2) RMS value of difference between iterated and actual "as measured" band levels over all 24 bands (no. of iterations in parentheses).

(3) PNL for the actual "as measured" spectrum at 300 m.

listed) assuming a real filter. Then, the value that would have been measured with an ideal filter is shown relative to this first value. Three of the 22 cases considered show an effective filter error in PNL, Δ_F (PNL), of greater than 0.2 dB. For the rest, this "error" is less than 0.1 dB. At first, this is surprising, considering the demonstrated magnitude of the filter errors Δ_F in individual bands for these and other spectra shown earlier. However, the explanation seems to be primarily that an individual band level Δ_F is large only when the band level itself is already low - too low to contribute significantly to PNL.

4.2 Filter Errors in EPNL Values

Two representative fly-by time histories, selected from the data defined in Volume I,[1] were selected for processing with the spectrum iteration algorithm, along with other methods for fly-by spectral analysis which do not account for filter errors but do attempt to account for missing high frequency bands due to excessive ambient background noise by various techniques. Results of this trial are summarized in Table 5. Again, it is clear that EPNL values, for which filter errors were eliminated, listed on the first line of Table 5, are not substantially different from the values computed by the other techniques. Each of these techniques are defined below.

Reference Method

This method was used to calculate ideal "Reference" spectral time histories. The procedure was:

1. For those bands within 5 dB of the acoustic ambient, energy subtraction of the ambient from the signal was performed. For the approach case (flight #18), a total of 5 bands were modified using this method. For #35, none were modified.

2. For those bands within 5 dB of an electrical noise floor, corrections were made using linear extrapolation from adjacent frequency bands (within the same time interval that the ambient violation occurred). For flight #18, two bands were modified using this method. For flight #35, seven bands were modified using this method. The frequency extrapolation was performed on "normalized" spectral time histories. These consist of spectra corrected, for inverse square law and air absorption losses,[16] to a constant propagation distance of

Table 5

Comparison of EPNL Values from Two Flights Computed by Employing Spectrum Iteration Technique and Other Conventional Methods

Method Used		EPNL Approach(3) (#18)	EPNL Takeoff(3) (#35)
Reference	With Spectrum Iteration(4)	104.19	115.64
	Without Spectrum Iteration	104.20	114.90
FAR Part 36	Within 5 dB(1)	104.18	114.99
	Within 2 dB	104.21	105.01
Baseline (Wyle #1)(2)	Up to 4 Bands/Interval	104.25	115.03
	Up to 8 Bands/Interval	104.29	115.10
Refined (Wyle #2)	Up to 4 Bands/Interval	104.24	114.89
	Up to 8 Bands/Interval	104.14	114.87

(1) Margin between background noise and signal for application of band elimination per Volume II.[2]

(2) EPNL values computed as defined in Volume I.[1]

(3) Measured fly-by time histories identified in Volume I.[1]

(4) The EPNL values computed with this method were based on "as measured" spectra corrected, for finite filter bandwidth and sideband errors, by the spectrum iteration method described in Section 3 and corrected for ambient level violations by the method defined in the text.

60 m (197 ft) for each 1/2-second time sample. The "normalized" spectra, including the extrapolated band levels, were then "unnormalized" back to the "as measured" distance before computing EPNL values.

Actually, two versions of this Reference method were used. The first method, identified on the first line, removed the filter errors by employing the spectrum iteration technique defined in Section 3 on the "as measured" spectra before making the additional corrections for ambient level. The second Reference method simply omitted the spectrum iteration correction so filter errors remain.

FAR Part 36

This method specified that all bands within 5 dB or 2 dB (two different runs) were to be deleted from the PNL calculation and hence EPNL calculation.

Baseline Method (Wyle Method #1)

This method specified that all (high frequency) bands within "X" dB of the ambient level (electrical ambient) be extrapolated at a rate of -9 dB/octave. "X" was chosen such that the maximum number of violating bands per time interval did not exceed either four or, for the second run, eight. For example, for the approach case (flight #18), to obtain no more than four violations/time interval, a signal-to-noise ratio of 16 dB was used as the violation criteria. Bands affected by an acoustic ambient were not modified.

Refined Method (Wyle Method #2)

This method was a more complex form of the "Baseline" Method. SPL values were first "normalized" to a distance of 60 meters for each 1/2-second sample of the spectral time history, for standard day weather conditions, and then missing bands filled in by extrapolation, at a rate of -6 dB/octave for the takeoff case and 0 dB/octave for the approach case. Acoustic ambient violators were unaffected. The "corrected, normalized" levels were then "unnormalized" by converting band levels back to "as measured" distances in order to compute PNL and EPNL values for standard day conditions.

4.3 Evaluation of Background Noise Effects

Further details on the potential compounding effect of background noise on filter errors is discussed in detail in Appendix A for idealized spectra and the reader is referred to that section for details and conclusions.

5. CONCLUSIONS

The results of this study of potential errors in PNL or EPNL values due to finite energy passed by filter skirts has demonstrated the following:

1. For the test cases evaluated, most showed a very small error (less than 0.1 dB) due to finite filter effects.
2. However, a few cases showed an error in PNL of about 1.1 dB.
3. The spectrum iteration scheme developed herein has proven to be a valuable tool for evaluating filter effects and is a promising means of eliminating or minimizing the substantial filter errors in individual bands which were documented in many of the cases considered.

Thus, as an improved tool for critical diagnosis of aircraft noise signatures, this scheme may have utility in the industry. Further refinements in details of the algorithm may be desirable before it should be universally adopted.

REFERENCES

1. Rackl, R., "Correction Procedures for Aircraft Noise Data, Volume I: Pseudotones," U. S. Department of Transportation, Federal Aviation Administration, Report No. FAA-EE-80-1, Volume I, May 1979.

2. Sutherland, L., Parkinson, J., and Hoy, D., "Correction Procedures for Aircraft Noise Data, Volume II: Background Noise Considerations," U. S. Department of Transportation, Federal Aviation Administration, Report No. FAA-EE-80-1, Vol. II, September 1979.

3. May, D. N. and Watson, E. E., "Correction Procedures for Aircraft Noise Data, Volume IV: Tone Perception," U. S. Department of Transportation, Report No. FAA-EE-80-1, Vol. IV, February 1980.

4. Brown, D. and Sutherland, L. C., "Correction Procedures for Aircraft Noise Data, Volume V: Propeller Aircraft Noise," U. S. Department of Transportation, Federal Aviation Administration, Report No. FAA-EE-80-1, Vol. V, February 1980.

5. U. S. Department of Transportation, Federal Aviation Administration, Federal Aviation Regulations, Part 36, "Noise Standards: Aircraft Type and Air Worthiness Certification," through Change 12, effective 15 January 1979.

6. International Electrotechnical Commission, "Electro-acoustical Measuring Equipment for Aircraft Noise Certification," IEC 561, Geneva, Switzerland, 1976.

7. International Electrotechnical Commission, "Octave, Half-Octave and Third Octave Band Filters Intended for the Analysis of Sounds and Vibrations," IEC 225, Geneva, Switzerland, 1966.

8. Society of Automotive Engineers, Inc., "Airplane Flyover Noise Analysis System Used for Effective Perceived Noise Level Computations," Aerospace Recommended Practice ARP 1264 (Revision in Process, 1980).

9. Shields, F. D. and Bass, H. E., "A Study of Atmospheric Absorption of High Frequency Noise and Application To Fractional-Octave Bands," NASA CR-2760, National Aeronautics and Space Administration, NASA-Lewis Research Center, Cleveland, OH, 1976.

10. Marsh, A., "Atmospheric-Absorption Adjustment Procedure for Aircraft Flyover Noise Measurements," U. S. Department of Transportation, Federal Aviation Administration, Report No. FAA-RD-77-167, December 1977.

11. American National Standards Institute, "Method for the Calculation of the Absorption of Sound by the Atmosphere," ANSI S1.26, New York, NY, 1978.

12. Engineering Sciences Data Unit, "Evaluation of the Attenuation of Broad-Band Sound by a Non-Uniform Still Atmosphere," ESDU Item No. 78003, London, England, September 1978.

REFERENCES (Continued)

13. Montegani, F. J., "Computation of Atmospheric Attenuation of Sound for the Fractional-Octave Bands," NASA Technical Paper 1412, National Aeronautics and Space Administration, NASA-Lewis Research Center, Cleveland, OH, February 1979.

14. Sutherland, L. C. and Bass, H. E., "Influence of Atmospheric Absorption on the Propagation of Bands of Noise," J. Acoust. Soc. Am. 66, 885-894, 1979.

15. Marsh, A. H., "Evaluation of Alternative Procedures for Atmospheric Absorption Adjustments During Noise Certification, Volume I: Analyses and Results," U. S. Department of Transportation, Federal Aviation Administration, Report under Contract DOT-FA78WA-4121, to be published 1980.

16. Society of Automotive Engineers, Inc., "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity," Aerospace Recommended Practice, ARP 866A, isued August 1964, reissued March 1975, Warrendale, PA, 1975.

17. Beranek, L. L., Acoustic Measurements, John Wiley and Sons, New York, NY, 1949.

18. Sepmeyer, L. W., "Bandwidth Error of Symmetrical Bandpass Filters Used for the Analysis of Noise and Vibration," J. Acoust. Soc. Am. 34, 1653-1657, 1962.

19. Sepmeyer, L. W., "On the Bandwidth Error of Butterworth Bandpass Filters," J. Acoust. Soc. Am. 35, 404-405, 1963.

20. American National Standards Institute, "Specifications for Octave, Half-Octave, and Third-Octave Band Filter Sets," ANSI Standard S1.11-1966(R-1971), New York, NY.

21. Society of Automotive Engineers, Inc., "Definitons and Procedures for Computing the Perceived Noise Lefel of Aircraft Noise," Aerospace Recommended Practice, ARP 865A, issued October 1964, reissued August 1969, Warrendale, PA, 1969.

APPENDIX A

SIMPLIFIED MODEL FOR SPECTRAL ANALYSIS OF BROADBAND NOISE CONSIDERING BOTH SIGNAL AND BACKGROUND NOISE INPUT COMPONENTS

1. Introduction

The following simplified analysis is carried out to provide an overall view of trends in the measurement of aircraft noise when effects of both finite filter slopes and background noise are considered. As defined later, and as indicated in Figure A-1, the spectral densities of the signal and noise elements are all assumed to have slopes defined by constant exponent relationships.

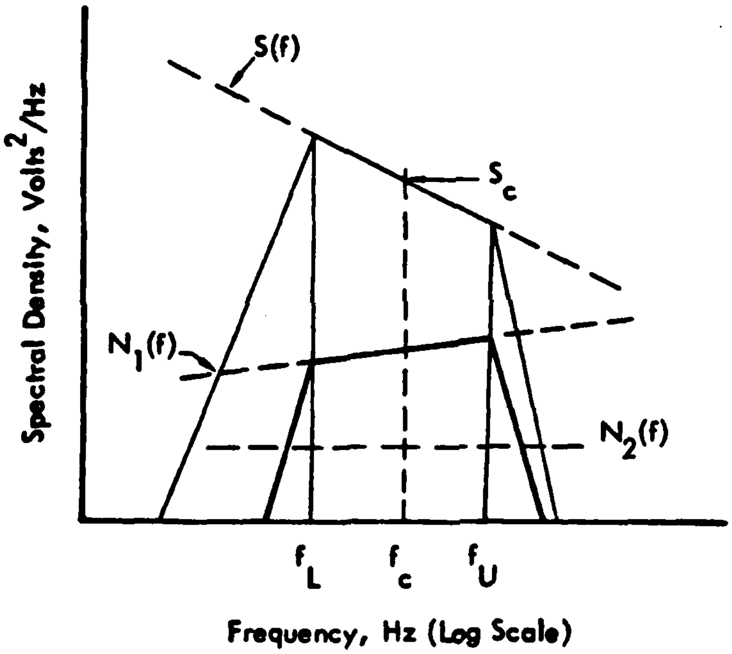


Figure A-1. Signal and Background Noise Spectral Density

The signal and two background noise elements are introduced into the simplified model of a spectral analyzer system in the way illustrated in Figure A-2.

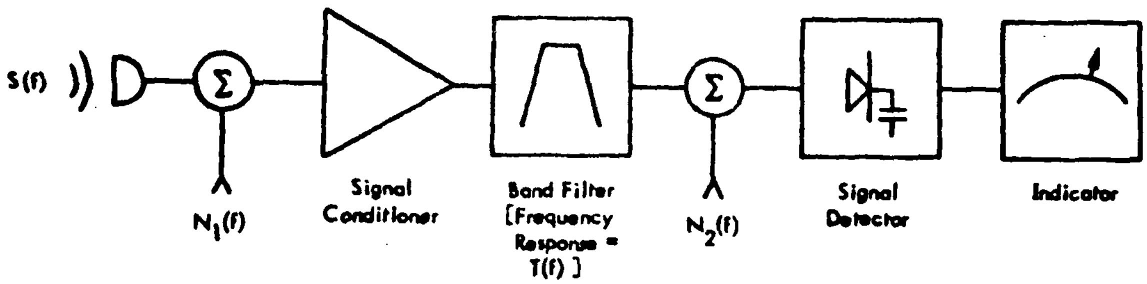


Figure A-2. Conceptual Diagram of Sound Measurement System Illustrating Introduction of Signal, S(f), Pre-filter Background Noise, $N_1(f)$, and Post-Filter Background Noise, $N_2(f)$, into Measurement Chain

All elements of the system are assumed to have unity gain over the nominal pass band of the spectrum analysis filter. The noise and signal components are also assumed to be uncorrelated. Thus, the signal power observed at the indicator (assuming the post-detection noise floor of the system is well below the measured signal and background noise) will be

$$P(f) = \left[S(f) + N_1(f)\right] \left[T(f) \quad + N_2(f)\right]$$

It should be noted that the background noise component defined, in Figure A-2, as post-filter background noise $N_2(f)$ is essentially the same as specified on pages 4 and 5 of Volume II in this report series, as "fixed electric background noise" $(N_{E2})^2$. Good design practice in any spectrum analyzer would dictate the need to strongly suppress this background noise component that is added to the signal between the filter output and detector. This becomes clear when one recalls that the total power of this post-filter noise component is the integral of its noise spectral density over the entire frequency range (i.e., greater than 50 to 10,000 Hz) of the analyzer while the power in the signal alone is roughly equal to the integral of its spectral density over just the filter bandwidth. Hence, to avoid contaminating the filtered signal level, the spectral density of the post-filter noise ($N_2(f)$ in Figure A-1 or A-2 must be substantially lower than the filtered signal spectral density, $S(f) \cdot T(f)$. The evaluation in this appendix will make this clear, but in this analysis of background noise effects, emphasis is placed on what is defined here as pre-filter background noise.

Based on the simple model defined in Figure A-2, the following defines the frequency variation for the signal and background noise spectra and the band pass filter response. The latter is a very simplified version of the frequency response of a real band pass filter; this simplification was chosen for convenience in the analysis that follows. Therefore, as a result of this simplified model for the filter response, we will be primarily concerned with <u>relative</u> changes in the measured band level due to the introduction of pre- or post-filter background noise. These relative changes in output are considered reasonable estimates of the true change in measured level when both background noise and finite filter slopes are considered.

Let $S(f)$ = Spectral Density of True Signal $= S_c(f/f_c)^m$ (A-1)

$N_1(f)$ = Spectral Density of Pre-filter Background Noise $= N_1(f/f_c)^n$

$N_2(f)$ = Spectral Density of Post-filter Background Noise $= N_2(f/f_c)^p$

$T(f)$ = Transmission Response of Filter $= \begin{cases} 1\cdot(f/f_L)^q & , f \quad f_L \\ 1 & , f_L \quad f \quad f_U \\ 1\cdot(f/f_U)^{-q} & , f \quad f_U \end{cases}$

where S_c, N_1 and N_2 = spectral densities of the signal and noise elements at the geometric center frequency f_c of the filter band in question.

A more convenient form for the spectral density slope exponents m, n, and p and the filter slope exponent $\pm q$ is provided by the corresponding slopes in dB per octave. This modified form can be defined as follows.

If the signal spectral density is defined, according to the first relationship above, by $S_c(f/f_c)^m$, the band power, $B(f)$, will be

$$B(f) = \int S(f)\,df = S_c f_c (f/f_c)^{m+1}/(m+1) \qquad \text{(A-2)}$$

and the corresponding band level, L_B, will be

$$L_B = 10 \log_{10}\left[B(f)/B_{ref}\right] = 10 \log_{10}\left[S_c f_c/(m+1)(B_{ref})\right] \quad \text{(A-3)}$$
$$+ 10(m+1)\log_{10}(f/f_c)$$

Thus, for any octave frequency interval, $(f/f_c) = 2$, so that the corresponding change, m', in band level over this frequency interval will be

$$m' = 10(m+1)\log_{10}(2) \quad , \text{dB/octave}$$

and the corresponding slope exponent quantity (m+1) becomes

$$m+1 = m'/10 \log_{10}(2) \quad \text{(A-4)}$$

The same relationship will hold for converting the band level slope, in dB/octave for the noise elements to their respective slope exponents, n and p.

A similar analysis method can be used to define the slope exponent for the filter side-band slopes - the only change is that no integration is involved so that, for a slope of q' dB/octave for filter skirts, the equivalent slope exponent q will be

$$q = \pm q'/10 \log_{10}(2) \quad \text{(A-5)}$$

For typical one-third octave band filters, q' can be very roughly approximated by a slope of 80 dB/octave.

2. Band Power Without Background Noise

A. Nominal Passband Power for Signal

Consider first the measured band levels without the influence of background noise. The power output B_I of the filter between the nominal filter cutoff frequencies f_L and f_U for a signal with a spectral density S(f) proportional to frequency raised to a constant power m can be defined by the integral:

$$B_I = \int_{f_L}^{f_U} S(f)\, df = S_c f_c \int_{f_L}^{f_U} (f/f_c)^m\, d(f/f_c)$$

$$B_I = \begin{cases} S_c f_c \left[(f_U/f_c)^{m+1} - (f_L/f_c)^{m+1}\right] \Big/ (m+1) & , m \neq -1 \\ S_c f_c \ln_e(f_U/f_L) & , m = -1 \end{cases} \quad \text{(A-6)}$$

Incorporating the relationships between f_L, f_U and f_c and, for convenience, expressing the total power relative to the power B_o out of the ideal filter with a constant spectral density, S_c input at the band center frequency f_c, eq. A-1 becomes

$$B_I = B_o\, C_I \tag{A-7}$$

where $B_o = S_c \left[f_U - f_L\right] = S_c \left[f_c\, r^{1/2} - f_c\, r^{-1/2}\right] = S_c f_c R$

$$C_I = \begin{cases} \left[r^{(1/2)(m+1)} - r^{-(1/2)(m+1)}\right] \Big/ (m+1)\,R & m \neq -1 \\ \text{or} & \\ \ln_e\left[r\right] \Big/ R & m = -1 \end{cases}$$

and $r = f_U/f_L, \quad R = r^{1/2} - r^{-1/2}$

B. Lower Sideband Power (B_L) for Signal

Following the same method, the power (B_L) in the lower sideband of the filtered signal can be defined as follows. (The lower sideband is assumed to be just 2 octaves wide extending from the lower band edge frequency f_L to $f_L/4$.)

$$B_L = \int_{f_L/4}^{f_L} S(f)\,T(f)\,df = S_c f_c \int_{f_L/4}^{f_L} (f/f_c)^m\,(f/f_c)^q\,(f_c/f_L)^q\,d(f/f_c) \tag{A-8}$$

$$= B_o\, C_L$$

where

$$C_L = \begin{cases} r^{(-1/2)(m+1)} \left[1 - (1/4)^{m+q+1}\right] \Big/ (m+q+1)\,R & , m+q \neq -1 \\ r^{q/2} \ln_e\left[4\right] / R & , m+q = -1 \end{cases}$$

$$R = r^{1/2} - r^{-1/2}$$

C. Upper Sideband Power (B_U) for Signal

Similarly, for the upper sideband, the signal power output B_U from a practical filter is assumed to be represented by

$$B_U = S_c f_c \int_{f_U}^{4f_U} (f/f_c)^m \, (f/f_c)^{-q} \, (f_c/f_U)^{-q} \, d(f/f_c) \qquad \text{(A-9)}$$

$$= B_o \, C_U$$

where

$$C_U = \begin{cases} r^{(1/2)(m+1)} \left[4^{m-q+1} - 1\right] \Big/ \left[(m-q+1)\, R\right] & , m-q \neq -1 \\ r^{q/2} \ln_e \left[4\right] / R & , m-q = -1 \end{cases}$$

3. Band Levels of Background Noise

A. Pre-Filter Noise

For the pre-filter background noise (i.e., background noise introduced into the system before the spectrum analysis filter), the observed band level can be defined in precisely the same way as for the signal. Assuming the noise spectral density is defined by $N_1(f) = N_1 (f/f_c)^n$, then equations A-7 to A-9 can be applied directly by simply substituting N_1 for S_c and n for m. Thus, the nominal pass band power B'_I, lower sideband power B'_L, and upper sideband power B'_U can be specified by

$$B'_I = B'_o \, C'_I \quad \text{, power of pre-filter noise within ideal passband} \qquad \text{(A-10a)}$$

$$B'_L = B'_o \, C'_L \quad \text{, power within lower sideband of practical filter} \qquad \text{(A-10b)}$$

$$B'_U = B'_o \, C'_U \quad \text{, power within upper sideband of practical filter} \qquad \text{(A-10c)}$$

where

$$B'_o = N_1 (f_U - f_L)$$

and C'_I, C'_L, C'_U are the same as defined for equations A-2, A-3, and A-4 with n substituted for m. Neglecting the special cases involving n = 1, etc.

$$C'_I = \left[r^{(\frac{1}{2})(n+1)} - r^{-(\frac{1}{2})(n+1)}\right] / \left[(n+1)R\right] \qquad , n \neq -1 \qquad \text{(A-11a)}$$

$$C'_L = r^{(-\frac{1}{2})(n+1)} \left[1 - (\tfrac{1}{4})^{(n+q+1)}\right] / \left[(n+q+1)R\right] \qquad , n+q \neq -1 \qquad \text{(A-11b)}$$

$$C'_U = r^{(\frac{1}{2})(n+1)} \left[4^{(n-q+1)} - 1\right] / \left[(n-q+1)R\right] \qquad , n-q \neq -1 \qquad \text{(A-11c)}$$

B. Post-Filter Background Noise

The background noise introduced after the filter is present in the measured output over the entire frequency spectrum. However, it will be assumed that for any one measured band, this component of background noise is present over 2 octaves on each side of the nominal passband of the one-third octave band filter. Clearly, this implies overlapping in the accounting for this noise element for adjacent passbands. However, this is exactly what would occur since each measured band level is considered to be the sum of all signal and background noise components present. Thus, the effective band power of this noise component, when measuring any one nominal band, will also be broken down into three parts as defined below. Again, we make use of the preceding equations A-10 and A-11 by substituting p for n and setting q = o to eliminate the filter.

$B''_I = B''_o C''_I$, power of post-filter noise within ideal passband (A-12a)

$B''_L = B''_o C''_L$, power within frequency range of lower sideband of practical filter (A-12b)

$B''_U = B''_o C''_o$, power within frequency range of upper sideband of practical filter (A-12c)

where

$$B''_o = N_2(f_U - f_L)$$

N_2 = Spectral density of post-filter noise at center band frequency f_c

$$C''_I = \left[r^{(\frac{1}{2})(p+1)} - r^{(-\frac{1}{2})(p+1)}\right] / \left[(p+1)R\right] \qquad , p \neq -1$$

$$C''_L = r^{(-\frac{1}{2})(p+1)} \left[1 - (\tfrac{1}{4})^{(p+1)}\right] / \left[(p+1)R\right] \qquad , p \neq -1$$

$$C''_U = r^{(\frac{1}{2})(p+1)} \left[4^{(p+1)} - 1\right] / \left[(p+1)R\right] \qquad , p \neq -1$$

4. Combined Signal and Background Noise

It is convenient to define the total power B_T, measured for just one band, in terms of the following matrix of band powers for the three regimes of the filter response (i.e., lower and upper sideband and ideal passband and the signal and noise elements in each regime.

Total Measured Power

	Lower Sideband	Ideal Passband	Upper Sideband	Total
Signal	$B_o C_L$	$\boxed{B_o C_I}$	$B_o C_U$	B_S
Pre-filter Noise	$B'_o C'_L$	$B'_o C'_I$	$B'_o C'_U$	B_{N_1}
Post-filter Noise	$B''_o C''_L$	$B''_o C''_I$	$B''_o C''_U$	B_{N_2}
Total	B_L	B_I	B_U	B_T

Only the outlined cell, $B_o C_I$, represents the power in the true signal within the ideal or nominal passband limits of the filter. All the other elements of this matrix represent unwanted added power segments which, under certain circumstances, can cause substantial errors to occur in the measurement of the desired true band power. Consider, now, some quantitative results which illustrate such situations.

A. Results

There are five parameters involved in portraying the results of this appendix - the slope exponents m', n' and p' for the signal and noise elements respectively and the relative levels N_1 and N_2 of the noise elements. Only a few of the many possible cases are considered here in detail. A summary table provides a brief overview of the trends over a wide range of all of these five variables.

Figure A-3 illustrates, for six specific cases, the relative levels in each of the cells in the preceding matrix of signal and noise elements which make up the total measured level. The cases illustrated cover combinations of signal and background

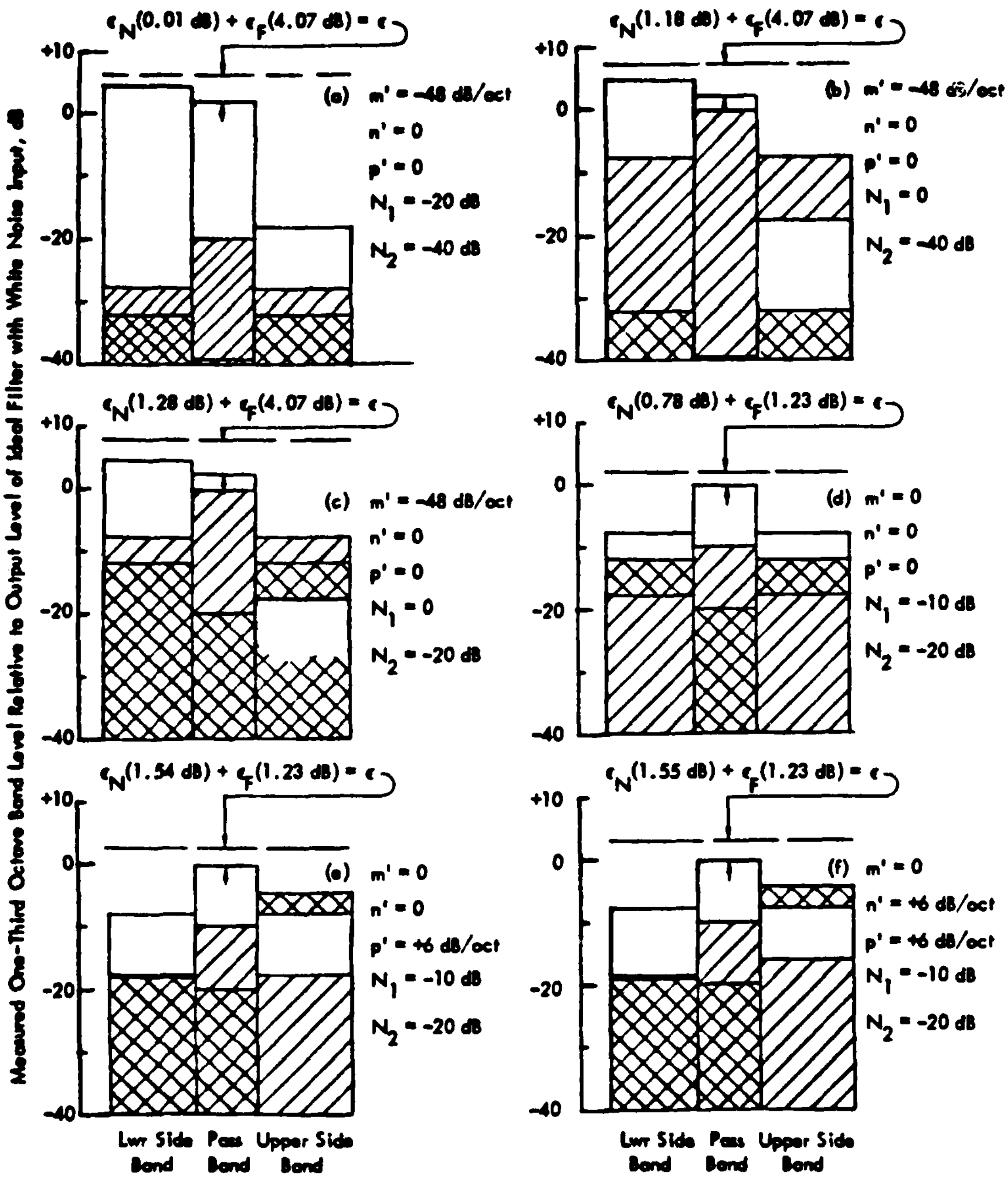


Figure A-3. Illustration of Computed Level for Each Element of Total Measured Output from a One-Third Octave Band Analyzer Based on the Simplified Model Defined in the Text.

The output components due to the signal ▭ ,the pre-filter noise ▨ , and the post-filter noise ▩ are given for several values of their respective slope and level parameters. The total measured level is signified by the dashed line on top of each figure. The total difference ϵ between this level and that due to the signal only within the ideal filter passband is the sum of the designated components ϵ_F, due to the added signal output from the filter sidebands and ϵ_N due to the background noise.

noise spectrum slope parameters of $m' = 0$ and -48 dB/octave, $n' = 0$ and $+6$ dB/octave, and $p' = 0$ and $+6$ dB/octave, respectively. Values of the noise level parameters are $N_1 = 0, -10$ and -20 dB and $N_2 = -20$ and -40 dB. The center bar represents the contribution to the measured level within the ideal filter passband for the signal and both noise elements (i.e., the pre- and post-filter noise), while the bars on either side present the corresponding outputs of the lower and upper filter sidebands (or the additional ± 2 octave range included in the post-filter background noise). The reference or zero level for each of these elements of the total measured output is the output level of an ideal filter for a white noise input. The total measured level is designated by the dashed line at the top.

For example, for case a, with $m' = -48$ dB/octave, the portion of the measured output contributed by the signal within the lower filter sideband is about 4 dB above the reference level; the corresponding passband level for the signal is 2.2 dB above the reference level, and the signal output from the upper sideband is -18 dB below the reference level. (Again, it is emphasized that these values are expected to have the correct relative magnitude but are not necessarily representative of actual absolute values due to the simple model chosen for the filter response.) In this case, the large negative slope of the signal input spectrum causes the lower sideband output of the filter to exceed the actual passband signal output by nearly 2 dB. The corresponding filter outputs for the pre- and post-filter noise, indicated by the single- and double-shaded bars, are symmetrical about the passband outputs, due to their zero band level slope, and the levels are well below the corresponding signal output as expected for their relative spectrum levels of $N_1 = -20$ and $N_2 = -40$ dB respectively.

The difference ϵ between the total measured level (signified by the dashed line) and the ideal signal band level within the filter passband is the sum of the designated error components, ϵ_N, due to the presence of the background noise and ϵ_F, due to the added signal output from the filter sidebands. While this latter error component is generally larger than will be found in actual filters due to the simplified model employed for the filter response, it is only shown here to provide a benchmark for evaluating the additional error ϵ_N due to the presence of pre-and post-filter background noise.

For example, consider cases (d) and (e), which differ only in the slope of the most-filter background noise spectrum. In each case, the spectrum level of this noise is 20 dB below that of the signal at the band center frequency (i.e., N_2 = -20 dB), but when the band slope of this noise component is increased from 0 to +6 dB/octave, the predicted error ϵ_N due to the presence of background nose increases by about 0.8 dB to 1.54 dB. Clearly, the requirement that post-filter background noise be insignificant suggests that its spectrum level be substantially more than 20 dB below that of the signal level.

Table A-1 summarizes the values of the error ϵ_N, as defined above, for values of the signal slope parameters m' and n' of 0, -24 and -48 dB/octave and 0 and -6 dB/octave respectively as well as for values of the pre-filter noise level parameter N_1 of 0, -5, -10, and -20 dB. As suggested at the beginning of this appendix, good design practice for a spectrum analyzer would dictate that the post-filter noise would be suppressed sufficiently to be entirely negligible so it has been ignored in this final summary.

The principal conclusions that can be drawn for this appendix are as follows:

1. Assuming the post-filter background noise is negligible, the error, ϵ_N, due to the remaining background noise, is defined by the difference in energy between the total measured signal and the background noise alone.

2. When the signal and pre-filter spectrum slopes are the same (i.e., both zero), the background noise error ϵ_N is simply equal to the usual increment in level when two uncorrelated signals are added on an energy basis.

3. The background noise error ϵ_N <u>decreases substantially</u> when the signal has a steep negative spectrum slope - a condition that will occur when analyzing high frequency bands of aircraft noise measured at relatively long distances and/or in conditions of high air absorption.

4. To avoid significant errors in the measured band level of a signal, due to background noise (i.e., $\epsilon_N < 0.1$ dB), the spectrum level of the pre-filter background noise should be at least 20 dB below that of the signal at the band center frequency. For practical purposes, in aircraft noise analysis,

Table A-1

Calculated Error, ϵ_N, in dB, in Total Measured Band Level
Due to Presence of Background Noise Introduced
Prior to One-Third Octave Band Analyzer Filter

m', Slope of Signal Band Level dB/Octave	N_1, Noise Spectrum Level* dB	ϵ_N, dB n', Slope of Noise Band Level dB/Octave -6	0
-48	-20	0.01	0.01
	-10	0.13	0.13
	-5	0.41	0.41
	0	1.20	1.18
-24	-20	0.03	0.03
	-10	0.32	0.31
	-5	0.93	0.92
	0	2.45	2.42
0	-20	0.04	0.04**
	-10	0.42	0.41**
	-5	1.21	1.19**
	0	3.05	3.01**

*Spectrum level of pre-filter noise at center frequency of band relative to signal spectrum level.

**Equal to normal correction for energy addition of two uncorrelated signals.

this translates to a predictable requirement that the aircraft signal to background noise ratio should be ... least 20 dB in each one-third octave band. This is an expected result which certainly did not require the preceding analysis to establish it. However, when this signal-to-noise ratio cannot be met, as is often the case in analysis of high frequency bands for aircraft noise, then one can conclude from the preceding the following significant and not immediately obvious conclusions.

5. The background noise error ϵ_N is nearly independent of the slope of the pre-filter background noise spectrum over the range of slopes (0 to ± 6 dB/octave) likely to be encountered. (As shown in Volume II of this report series,[2] the slope of the acoustical background noise is typically approximately -4.5 dB/octave while the spectrum slope of the pre-filter background noise (or electric background noise in the parlance of Volume II)[2] can vary from at least -3 to +3 dB/octave.)

6. Thus, the total band level of the signal, <u>including</u> the power in the filter sidebands but <u>excluding</u> any background noise contribution can be determined by simply subtracting, on an energy basis, the pre-filter (or predetection) background noise band level from the total measured band level. This noise-free signal band level can now be used with the spectrum iteration procedure outlined in Section 3 of the main text to determine the approximate shape of the true spectral density of the signal thus enabling one to compute true signal band levels and/or adjust the signal levels for other distance or weather conditions and eliminate the measurement error due to finite filter sidebands in the presence of moderate pre-filter background noise.

7. To insure that this last procedure is carried out correctly for all types of signals contaminated by pre-filter background noise, it may be desirable to actually process the "as measured" signal through the spectrum iteration routine first in order to accurately define the true "signal + noise spectrum. Then, by applying an energy subtraction of the true noise spectrum, one obtains a true noise-free signal spectrum without further iteration which is ready for applying frequency-sensitive corrections and/or computing true band levels simulating ideal filters. (See Section 3 for a further discussion of this concept.)

APPENDIX B

INFLUENCE OF ATMOSPHERIC ABSORPTION ON THE PROPAGATION OF BANDS OF NOISE

By

L. C. Sutherland

H. E. Bass

Reprint of Paper Published in
Journal of Acoustical Society of America

September 1979

ERRATA

<u>Page</u>

B-1 <u>Abstract</u> - Fourth line from bottom; change "4 to 40" to read "4 and 10"

B-2 <u>Section 1.</u> - Third line after Eq.(2); change to read "transmission loss of practical filter ≈ $f_1/5$ and $5 \cdot f_2$, respectively . . ."

B-6 Eq. (26) The first bracketed term should read $10 \log \left[\sum_{j}^{nb} B'_{rj} \right]$

B-7 Figure 5 and Figure 6. In titles, substitute $\Delta_F(S)$ for $\Delta_f(S)$ and $\Delta_F(R)$ for $\Delta_f(R)$, respectively

B-9 <u>Section V.</u> - Fourth line from bottom; change "distance" to "distant"

B-10 <u>Reference 16</u> - Should read "American National Standard S1.11-1966(R-1971). . ."

Influence of atmospheric absorption on the propagation of bands of noise

L. C. Sutherland

Wyle Laboratories, El Segundo, California 90245

H. E. Bass

Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677

(Received 11 January 1979; accepted for publication 30 April 1979)

As a band of noise propagates through the atmosphere, atmospheric absorption attenuates the higher frequencies in the band more rapidly than the lower frequencies. For large propagation distances, atmospheric absorption shapes the spectrum such that most of the acoustic energy is in the lower part of the band. When this occurs, the effective atmospheric absorption coefficient for this band of noise is less than when the energy is evenly distributed over the band. When a real filter is used to examine the received spectrum, the problem becomes even more complex since the filter does not cut off completely at the nominal lower-frequency limit for the band. The equations governing the attenuation and spectrum analysis of noise under these conditions have been derived and a numerical integration process has been used to obtain quantitative results. It is shown that predicting observed values of atmospheric absorption loss for bands of noise involves two correction factors which are added to single-frequency absorption loss to define the effective attenuation for the band of noise. The first correction factor Δ_l accounts for the difference between the attenuation predicted over a given path in a homogeneous atmosphere for a pure tone at the geometric center frequency of an ideal filter band, and the actual attenuation over the same path for the same ideal band of noise. The second correction factor Δ_F accounts for the difference between the band level measured at one point with a practical filter with finite transmission outside the nominal filter pass band and the true band level measured at the same point with an ideal (rectangular) filter. Representative calculations show that the sum of these corrections become substantial when the product of distance, in km, and the square of frequency, in kHz, exceeds about 4 to 40 for octave and one-third octave band filters, respectively. Although the net effect of such corrections will generally be small for overall frequency weighted-noise levels (e.g., perceived noise level), the errors in diagnosing spectral detail in noise signatures of distant sources, such as aircraft, can be important.

PACS numbers: 43.28.Fp, 43.50.Vt

INTRODUCTION

Recent experimental[1-3] and theoretical[4-5] work have made it possible to compute,[6] with high precision, the acoustic absorption of a pure tone as it propagates through a still atmosphere free from the influence of a ground plane.[7] Although there are still some unresolved questions concerning the contribution to the attenuation of the signal by turbulence,[8] radiation,[9] dust,[10] fog,[11] or finite amplitude effects,[12] the major contributions to the pure tone absorption coefficient are now well understood and documented. In practice, however, one is seldom confronted with the propagation of pure tones. Instead, the source of interest typically generates some type of noise spectrum with energy spread continuously over a range of frequencies. It is the absorption of such broadband noise, then, rather than of a pure tone, that is more frequently of practical interest, particularly for the analysis of air to ground propagation for aircraft noise.

Atmospheric absorption is an increasing function of frequency. As a result, when a band of noise propagates through the atmosphere, the higher frequencies in the band are attenuated most strongly. If the spectrum is initially flat, after propagating some distance, it will begin to exhibit greater and greater attenuation at high frequencies as either distance or frequency increases. This spectrum-shaping effect of atmospheric absorption compounds the problem of spectral measurements of broadband noise signals which propagate over long distances in the atmosphere.

The objective of this paper is to outline the errors that may occur and outline appropriate correction procedures when one evaluates the atmospheric absorption loss of bands of noise in terms of the pure tone absorption coefficient $\alpha(f_c)$ at the center frequency f_c of a band. As outlined in Fig. 1, two basic types of errors are involved. The first, designated by the symbol Δ_l, is the difference, in decibels, over a fixed path of length l in a homogeneous atmosphere, between the absorption loss for the pure tone corresponding to the center frequency of a band of noise and the loss for the total power within the same band of noise as measured with an ideal filter. The second type of error, designated by the general symbol Δ_F, equals the difference, in decibels, between the total power passed by a practical filter and the power passed by its nominal (ideal) pass band.[13,14] The first correction factor, Δ_l, will vary with the nominal filter bandwidth and the slope of the source or receiver spectrum, when analyzing propagation loss from a source to a receiver, or from a receiver back to a source. In addition, Δ_l will vary with the atmospheric conditions over the propagation path and, unlike the pure tone absorption coefficient $\alpha(f_c)$ at the band center frequency, Δ_l will also vary with distance along the propagation path. The second correction factor, Δ_F, can be defined, completely, in terms of only the frequency response of the filter and upon the spectrum

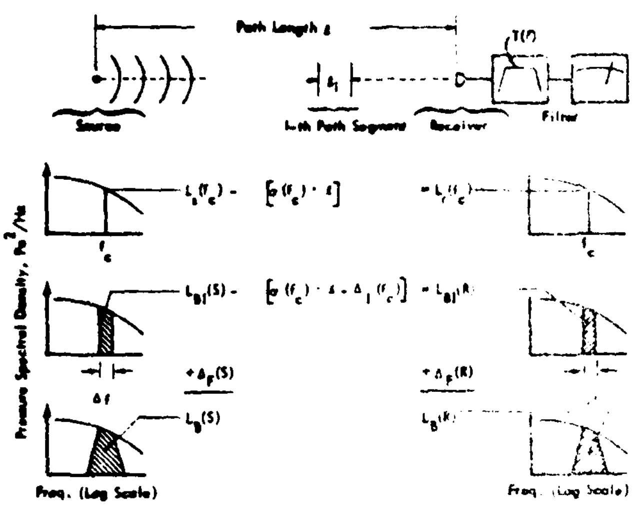


FIG. 1. Illustration of the application of two correction factors, Δ_l and Δ_F, for analyzing propagation loss of bands of noise. Δ_l is the difference between the pure tone loss $[\alpha(f_c)\cdot l]$ at the band center frequency f_c over a path length l between a source (S) and receiver (R), and the loss over the same path for an ideal band of noise centered on the same frequency. Δ_F is the difference in decibels between the power level L_B transmitted by an actual filter with finite transmission outside the nominal passband and the power level L_{BI} transmitted by an ideal (rectangular response) filter centered on the same frequency.

shape of its input signal. However, the spectrum shape of the filter input signal *at a receiver* is also influenced by the same atmospheric and path length factors that influence the ideal noise band to pure tone correction factor Δ_l. Thus, in many cases, it is convenient to compute one composite correction term, Δ, which varies with all of the parameters just defined.

The fact that atmospheric absorption will shape the spectrum within a band of noise has been well recognized for some time. For example, the SAE standard on air absorption[15] requires that at frequencies above 4 kHz, the pure tone absorption coefficient at the lower band edge frequency should be used to compute effective absorption coefficients for octave bands, while at lower frequencies, the center frequency of the band should be used. While this is a reasonable rule of thumb, considering typical aircraft noise spectra, it can lead to significant errors.[1]

This paper will present a mathematical representation for the propagation of a band of noise through the atmosphere and numerical procedures for defining the correction factors Δ_l and Δ_F which yield quantitative results not dependent on approximations such as used in the SAE standard. The numerical procedure is used to compute band loss coefficients for a few typical cases. Due to the many variables involved, including weather, initial spectrum shape, type of filter, and distance, only a few limited cases can be presented within the scope of this paper. Condensed tables are also available for a few specific cases.[3,4]

I. METHOD OF ANALYSIS

In general, the analytical methods used to define the correction factors Δ_l and Δ_F involve the computation, by integration over a specified bandwidth, of the relative level (in decibels) of a broadband random signal with a nonuniform spectrum. The analysis does not attempt to include composite signals made up of broadband noise plus one or more pure tone components. However, such signals can be handled by a superposition process, applying the methods outlined herein for computing the absorption loss of the random noise portion and the simpler methods applicable for computing absorption losses of pure tones.[6]

The specific expression which defines Δ_l can be stated as follows (throughout this analysis, correction factors are derived for only one filter band but they can obviously be applied to any number of bands by the same techniques.)

$$\Delta_l = \alpha(f_c)\cdot l + 10\log_{10}\left(\int_{f_1}^{f_2} P_s^2(f)\cdot 10^{-\alpha(f)\cdot l/10}\,df \Big/ \int_{f_1}^{f_2} P_s^2(f)\,df\right), \tag{1}$$

where $\alpha(f_c)$ is the pure tone absorption coefficient at the band center frequency f_c, $\mathrm{dB\cdot m^{-1}}$; l is the propagation path length, m; $P_s^2(f)$ is the pressure spectral density, at source reference position, at frequency f, $\mathrm{Pa^2\cdot Hz^{-1}}$; and f_1, f_2 are the nominal lower and upper cutoff frequencies of ideal filter band centered on frequency f_c, Hz.

The filter correction factor $\Delta_F(R)$ at the receiver is defined by

$$\Delta_F(R) = 10\log\left[\int_{f_L}^{f_U} P_r^2(f)10^{-TL(f)/10}\,df \Big/ \int_{f_1}^{f_2} P_r^2(f)\cdot df\right],\ \mathrm{dB}, \tag{2}$$

where $TL(f)$ is the power transmission loss of filter, dB; f_L, f_U are the effective lower and upper limits of transmission of practical filter $\approx f_1/8$ and $8\cdot f_2$, respectively, Hz; and $P_r^2(f)$ is the pressure spectral density at input to receiver filter, $\mathrm{Pa^2\cdot Hz^{-1}}$.

A comparable expression can also be written for the filter correction factor $\Delta_F(S)$ at the source by substituting the subscript s for r in the preceding equation.

Since the integrands in these equations cannot, in general, be expressed in a simple, integrable closed form valid over the full width of, say, a $\frac{1}{3}$-octave band filter, it is necessary to resort to a process of numerical integration. The usual case of interest is for constant percentage filters for which the ratio of the upper and lower nominal band edge frequencies f_2 and f_1 can be specified as $r = f_2/f_1$ and the geometric center frequency is $f_c = (f_1 f_2)^{1/2}$. First, the bandwidth $f_2 - f_1$ of the actual filter is divided into b constant percentage rectangular filter elements. For the jth element, the ratio of the upper band edge frequency f_{j+1} to the lower band edge frequency f_j is equal to $r^{1/b}$ and the frequency f_j is equal to

$$f_j = f_c(r^{-1/2})[r^{(j-1)/b}],\ \mathrm{Hz} \tag{3}$$

where j varies from 1 at the lower band edge of the entire filter to $(b+1)$ at the upper band edge. Examine, now, the application of this numerical integration approach.

II. DERIVATION OF Δ_l

Consider first, the integral for the power in an ideal filter band at the source as specified by the denominator of Eq. (1). If the bandwidth $f_{j+1}-f_j$ of the jth element within this ideal filter band is sufficiently small, it is possible to assume that, over this narrow frequency range, the pressure spectral density at the source can be specified by a simple power law expression of the form

$$P_s^2(f)=P_{sj}^2(f/f_j)^{m_j}, \quad \mathrm{Pa}^2\cdot\mathrm{Hz}^{-1}, \tag{4}$$

where $P_{sj}^2=P_s^2(f_j)$, the pressure spectral density at the lower edge $(f=f_j)$ at the jth element, and m_j equals the exponent which defines the slope of the spectral density over the frequency range f_j-f_{j+1}. This simple power law is equivalent to assuming that a log–log plot of the spectral density versus frequency is a straight line with a constant slope, over the frequency interval f_j to f_{j+1}, defined by the exponent m_j. The total power B_{sj} within this jth elemental band, at the source (corresponding to the mean square pressure in the source signal), is given by the integral of $P_s^2(f)$ over the same frequency interval. Thus, with Eq. (4), one has

$$B_{sj}=\int_{f_j}^{f_{j+1}}P_s^2(f)\,df=P_{sj}^2\int_{f_j}^{f_{j+1}}(f/f_j)^{m_j}\,df, \quad \mathrm{Pa}^2. \tag{5}$$

With Eq. (3), this integral for the total power in the jth filter element at the receiver reduces to

$$B_{sj}=\begin{cases}P_{sj}^2f_j(r^{(m_j+1)/b}-1)/(m_j+1), & \mathrm{Pa}^2, \quad m_j\neq-1,\\ P_{sj}^2f_j(\ln_e(r)/b, & \mathrm{Pa}^2, \quad m_j=-1.\end{cases} \tag{6a}$$

Summing over the entire band gives the total band power B_s at the source

$$B_s=\sum_{j=1}^{b}B_{sj}. \tag{6b}$$

The total air absorption loss over the source–receiver path l can also be given to a good approximation, over the same narrow frequency range f_j to f_{j+1}, by a power law of the form,

$$A(f)=A_j(f/f_j)^{-n_j}, \tag{7}$$

where $A_j=A(f_j)=10^{-\alpha(f_j)\cdot l/10}$, the fractional loss in power by absorption at the lower band edge frequency f_j of the jth filter element, and n_j is the exponent which defines, approximately, the change in total absorption with frequency over the narrow frequency range of this jth element. Then the pressure spectral density at the receiver $P_r^2(f)$ can be expressed, with Eqs. (4) and (6), as

$$P_r^2(f)=P_s^2(f)\cdot A(f)=P_{sj}^2(f/f_j)^{m_j}A_j(f/f_j)^{-n_j}, \quad \mathrm{Pa}^2\cdot\mathrm{Hz}^{-1}. \tag{8}$$

Thus, by comparison of Eqs. (4) and (8), we can use Eq. (6) to express the total power B_{rj} (or corresponding mean square pressure) in the jth elemental band at the receiver as:

$$B_{rj}=\begin{cases}P_{sj}^2A_jf_j[r^{(m_j-n_j+1)}-1]/(m_j-n_j+1), & m_j-n_j\neq-1, \quad (9a)\\ P_{sj}^2A_jf_j[\ln_e(r)]/b, & m_j-n_j=-1. \quad (9b)\end{cases}$$

Again summing over the entire band, the total band power at the receiver is

$$B_r=\sum_{j=1}^{b}B_{rj}. \tag{9c}$$

Each of the slope exponents m_j and n_j can be determined from their corresponding log–log relationships. For example, m_j is determined by setting the frequency $f=f_{j+1}$ in Eq. (4), taking the log of both sides, and solving for m_j to give

$$m_j=[\log_{10}P_s^2(f_{j+1})-\log_{10}P_s^2(f_j)]/\log_{10}(f_{j+1}/f_j). \tag{10}$$

Substituting the spectrum level $L_s(f)$ corresponding to the value of $10\log_{10}P_s^2(f)$ and using Eq. (3), this reduces to

$$m_j=[L_s(f_{j+1})-L_s(f_j)]/[(10/b)\log_{10}r]. \tag{11}$$

Applying the same process for n_j, using Eqs. (7) and (3),

$$n_j=[\alpha(f_{j+1})-\alpha(f_j)]\cdot l/[(10/b)\log_{10}r]. \tag{12}$$

The values of $\alpha(f)$ required in this last expression are computed with the methods of Ref. 6 at the two elemental filter frequencies f_j and f_{j+1} and for the average atmospheric conditions prevailing over the path length l. (Note that this path can, if necessary, be just one arbitrarily small incremental segment in a layered atmospheric model. A different correction factor Δ_l is then computed for each such elemental path segment and for each frequency band.)

The parameters b and r are preassigned constants. Based on the numerical integration method employed to sum powers over each filter band, the constant b (the number of integration elements per band) is an even integer and, to achieve acceptable accuracy (integration error <0.1 dB), it must be at least 12 for either $\frac{1}{3}$ or $\frac{1}{1}$ octave bands. The bandwidth ratio r is set equal to $10^{0.1}$ or 2 for $\frac{1}{3}$ or $\frac{1}{1}$ octave band filters, respectively.

All that remains in order to carry out the integration over all b segments in each filter band, is a means of specifying the spectral density or corresponding spectrum level at any frequency f within the overall filter bandwidth f_1-f_2. Two approaches have been employed as illustrated in Fig. 2. The first, and simplest, was to assume that the slope of the spectrum level curve is constant over the full nominal bandwidth of the filter under consideration. The second approach assumed that the spectrum level was given by a general interpolation or spline function $G(f)$ which defined a spectrum level curve which passes exactly through the true spectrum levels at the geometric center frequencies f_{i-1}, f_i, and f_{i+1}, of the ith band and the immediately adjacent bands.

A. Constant slope approach

If the spectral density of the source has a constant slope over the entire frequency bandwidth of a filter, then it can be defined by Eq. (4) letting P_{sj}^2 be the spectral density $P_s^2(f_1)$ at the lower cutoff frequency (f_1) of the filter and m_j becomes simply the constant slope m.

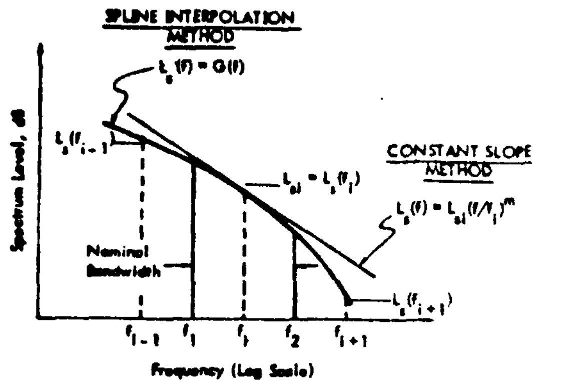


FIG. 2. Illustration of the two general methods considered to define the spectrum level $L_s(f)$ at any frequency for analysis of the propagation loss for the ith band of noise with center frequency f_i.

A more convenient definition of the spectrum shape is provided by the slope (S_B) of the filtered band levels in dB per bandwidth. Since the power within a constant percentage band increases directly with frequency to the first power for a *constant input spectral density*, if the spectral density varies with frequency to the power m, then the band power varies with frequency to the power $(m+1)$. Thus, if the ratio of upper to lower cutoff frequencies is r, then the slope S_B or change in band level, in decibels, over a frequency interval of just one bandwidth (for which the ratio of upper to lower frequencies is r) will simply be

$$S_B = 10\log_{10}(r)^{m+1} = 10(m+1)\log_{10} r, \quad \mathrm{dB}\cdot(\mathrm{band})^{-1}. \tag{13}$$

Following this same "constant slope" approach, assume that one can approximate the actual frequency variation in the atmospheric absorption over one entire filter bandwidth by the simple power law given by Eq. (7). Replacing A_j by the corresponding absorption loss A_1 at the lower cutoff frequency f_1 of the filter, then at any frequency within the band, the fractional loss due to absorption is

$$A(f) = A_1(f/f_1)^{-n}. \tag{14}$$

[This is, in fact, a rather crude approximation chosen here for simplicity in the analysis. A better approximation for the frequency variation in atmospheric absorption loss over the path l is equal to $\exp[-\alpha_1(f/f_1)^k \cdot l]$, where α_1 is the absorption coefficient (in nepers$\cdot$m^{-1} instead of dB$\cdot$m^{-1}) at the reference frequency f_1 and the exponent k need only vary from 0 to 2.] Again, a more convenient way to describe the variation in absorption as a function of frequency by this approximate form is given by the slope S_A or change, in decibels, of the absorption loss over a frequency interval of one filter bandwidth. Thus, the parameter S_A is given by

$$S_A = 10\log_{10}(r)^{-n} = -10n\log_{10} r, \quad \mathrm{dB}\cdot(\mathrm{band})^{-1}. \tag{15}$$

From Eq. (1), the correction factor Δ_l can be expressed in simpler form as

$$\Delta_l = \alpha(f_c)\cdot l + 10\log_{10}[B_r/B_s], \quad \mathrm{dB}, \tag{16}$$

where B_r and B_s are the band powers at the receiver and source respectively for ideal filters.

Further, by transforming the first term into decibel notation and employing Eq. (14), it can be expressed as

$$\alpha(f_c)\cdot l = -10\log_{10}[A(f_c)] = -10\log_{10}[A_1(f_c/f_1)^{-n}], \quad \mathrm{dB}, \tag{17}$$

where $f_c/f_1 = r^{1/2}$, the ratio of the geometric mean frequency (f_c) to the lower cutoff frequency (f_1) of the filter.

Now, by using Eqs. (6) and (9) to define the band powers B_s and B_r at the receiver and source, respectively [dropping the j subscript since we are now considering only one integration element over the entire filter bandwidth (i.e., $b=1$)], and including Eq. (17), Eq. (16) becomes, for the nonsingular case (i.e., $m \neq -1$ or $m-n \neq -1$)

$$\begin{aligned}\Delta_l &= 10\log_{10}[B_r/(B_s A_1 r^{-n/2})] \\ &= 10\log_{10}[(r^{m-n+1}-1)(m+1)/(r^{-n/2})(r^{m+1}-1)(m-n+1)]. \end{aligned} \tag{18}$$

Employing the identities $r^x - r^{-x} = 2\sinh(x\ln r)$ and $\log_{10} r/\ln r = \log_{10} e$, Eq. (18), after a bit of algebra and including Eq. (13) and (15), becomes

$$\Delta_l = 10\log_{10}\{S_B\sinh[\beta(S_A+S_B)]/(S_A+S_B)\cdot\sinh(\beta S_B)\} \ \mathrm{dB}, \tag{19a}$$

where $\beta = 1/20\log_{10} e = 0.11513$.

For the singular cases for which $m=-1$ or $m-n=-1$, the equivalent for Eq. (19) is that $S_B=0$ or $S_A+S_B=0$.

For these cases, it can be shown that, for $S_B=0$,

$$\Delta_l = 10\log_{10}[\sinh(\beta S_A)/\beta S_A], \quad \mathrm{dB}, \tag{19b}$$

and, for $S_A+S_B=0$,

$$\Delta_l = 10\log_{10}[\beta S_B/\sinh(\beta S_B)], \quad \mathrm{dB}. \tag{19c}$$

Thus, simple, though approximate, expressions are obtained in terms of just two variables, S_A and S_B, for the correction factor Δ_l which specifies the difference between the single frequency absorption loss and the true value for an ideal band centered on the same frequency. The slope S_A of the absorption loss can be estimated by computing the values for the total absorption loss at frequencies equal to one filter bandwidth below (f_c/r) and above ($f_c\cdot r$) the center frequency (f_c) of the filter in question and computing the average slope by

$$S_A \simeq [\alpha(f_c\cdot r) - \alpha(f_c/r)]\cdot l/2, \quad \mathrm{dB}\cdot(\mathrm{band})^{-1}. \tag{20}$$

The slope of the source spectrum S_B can be approximated in a similar fashion or estimated by graphical means.

The correction factor Δ_l for an ideal $\frac{1}{3}$-octave band filter, as defined by Eq. (19), is presented graphically in Fig. 3 as a function of S_B and S_A in the form of contours of constant values of Δ_l. While the values of Δ_l shown are only as accurate as the approximations employed in deriving Eq. (19), they provide a useful first approximation. There are several characteristics of

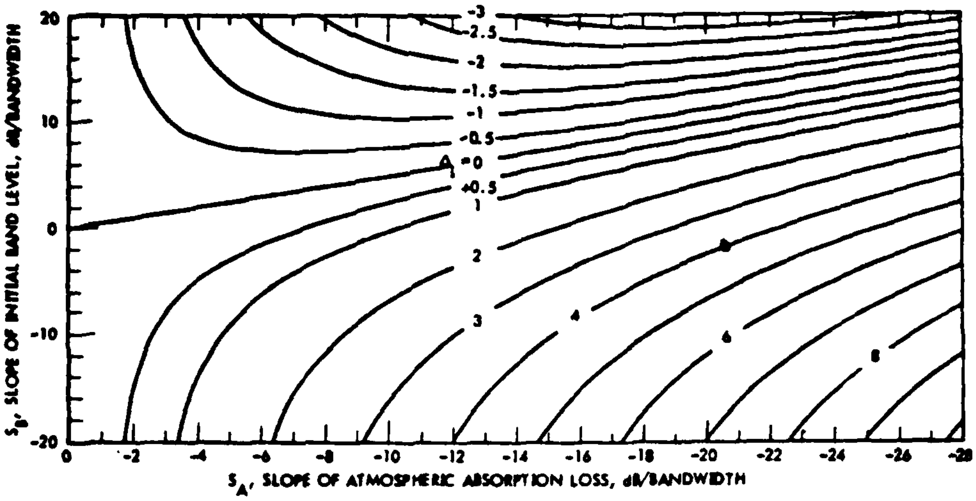


FIG. 3. Contours of the value of approximate correction factor (Δ_f).

Fig. 3 of interest. First, for typical conditions, the correction Δ_f is not large; it varies from about 6% to 8% of the total pure tone absorption loss $\alpha(f_i)\cdot l$, with the highest values occurring at the largest distances. Second, Δ_f is usually positive since for most full scale noise sources, the energy in a band, after propagating through the atmosphere, is concentrated at frequencies near the lower band edge where atmospheric absorption is lower than at the band center. Thus, the true absorption loss for the band is less than the pure tone loss of the band center frequency. Note that for this simplified approach, the values of Δ_f in Fig. 3 are independent of the filter bandwidth since the slope parameters S_A and S_B are in decibels per bandwidth. In fact, of course, the actual value of Δ_f for an octave band filter will be substantially greater than for a $\frac{1}{3}$-octave band filter for the *same* actual spectrum and absorption slopes in dB/octave. We can conclude that the pure tone absorption at the band center frequency is a reasonable rough approximation to the absorption encountered by a band of noise analyzed with a perfect filter provided the band is narrow, and the slopes of the initial source spectrum and the air absorption curve are not large. The latter constraint implies that the frequency should be low and the propagation path short. Specific quantitative guidelines as to when these conditions may be met will be given later.

B. Varying spectrum slope

If the spectral density $P_s^2(f)$ of the source has a varying slope over the bandwidth of the filter, an accurate computation of Δ_f for this case requires the use of some type of spline curve-fitting function for estimating $P_s^2(f_j)$ at each of the integration frequencies f_j over the filter bandwidth. Several methods could be employed for generating such spline functions including a standard cubic spline function for interpolating *between two points*. However, it was more convenient and sufficiently accurate to use a simple log-linear equation with a linearly varying slope to interpolate *between three points*—the center frequencies of three adjacent filter bands centered on any one band of interest. Thus, referring to Fig. 2, over the frequency range of ±1 filter band about the ith band, or from f_{i-1} to f_{i+1}, the spectrum level of the source $L_s(f)$ was assumed to vary as

$$L_s(f) = L_s(f_i) + 10[a_i + b_i(f/f_i)]\log_{10}(f/f_i)\,, \quad \text{dB}\,, \tag{21}$$

where $L_s(f_i)$ is the spectrum level at the center frequency (f_c) of the ith band, and a_i, b_i are the constants determined by the differences, δ_{i-} and δ_{i+}, between the spectrum levels of the three adjacent bands as shown by the following expressions:

$$a_i = -(\delta_{i+} - r^2\delta_{i-})/10(r^2-1)\log_{10} r\,,$$
$$b_i = (\delta_{i+} - \delta_{i-})/10(r^2-1)\log_{10} r\,, \tag{22a}$$

and

$$\delta_{i+} = L_s(f_{i+1}) - L_s(f_i)\,, \quad \text{dB}\,,$$
$$\delta_{i-} = L_s(f_i) - L_s(f_{i-1})\,, \quad \text{dB}\,. \tag{22b}$$

Note that when $\delta_{i+} = \delta_{i-}$, then $b_i = 0$, and Eq. (21) would describe the spectrum level curve of a linear plot of spectrum level versus log frequency with a constant slope of $10a_i$ dB/decade. However, the type of smooth curve depicted in Fig. 2 for the spline interpolation method is readily described by Eq. (21) for nonzero values of b_i. This more general approach for interpolating the spectral density at frequencies between adjacent band center frequencies was particularly useful when dealing with practical spectral shapes as discussed later. This was accomplished by using the varying slope concept specified by Eq. (21) to define the spectral density at the source, P_{sj}^2, at each jth frequency f_j as

$$P_{sj}^2 = P_{si}^2 (f_j/f_i)^{a_i + b_i(f_j/f_i)}\,, \tag{23}$$

where P_{si}^2 is the pressure spectral density at the center frequency f_i of the ith band, and a_i, b_i are the slope constants derived with Eq. (22) from the estimated spectrum levels of the source at each band center frequency. [The same techniques outlined here to compute Δ_f for the general case of a varying slope spectra can be used to determine the true relationship between source band

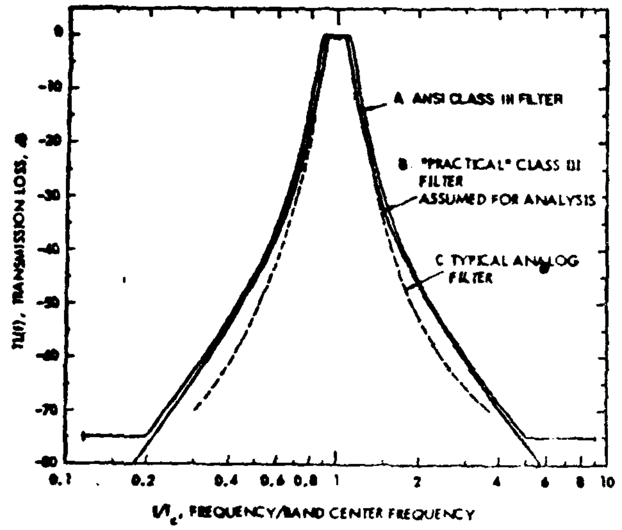


FIG. 4. Transmission loss curve (a) for a hypothetical "Practical" $\frac{1}{3}$-octave band filter compared to (b) the minimum required loss according to ANSI Standard S1.11 and (c) the actual transmission loss of a typical analog filter.

levels (assuming perfect filters) and corresponding spectrum levels at the band center frequencies without requiring the usual assumption of a flat spectrum over the filter bandwidths.]

Thus, the spectral density at the source can be computed at any frequency (within the range $f_{i-1}-f_{i+1}$) using Eq. (23) and the integration of Eq. (1) carried out using the numerical integration techniques specified by Eqs. (3)–(12).

The final expression for Δ_l for the case of the varying spectrum slope can now be given as

$$\Delta_l = \alpha(f_c)\cdot l + 10\log\left[\sum_j B_{rj}\right] - 10\log\left[\sum_j B_{sj}\right] \qquad (24)$$

where B_{rj}, B_{sj} are the powers in the jth element at the receiver and source, respectively, as defined by Eqs. (9) and (6). The corresponding values of P^2_{sj} are defined by Eq. (23). The energy sum of these elemental powers over the b segments defines the true band powers of the receiver and source, respectively, within the ideal filters.

III. DERIVATION OF Δ_F

When a real filter is used, the same numerical integration techniques just defined for the computation of Δ_l are again utilized. However, the integral in the numerator of the second term of Eq. (1) for Δ_l is modified in Eq. (2) in two ways. The integration limits are extended to cover the practical transmission range of the actual filter and the nonuniform power transmission loss of the filter is included within the integral.

Approximately $\frac{1}{5}$ and 5 times the band center frequency define reasonable limits for the practical transmission range of a filter. However, just one filter bandwidth below or above the filter band edge frequencies can often be used as effective limits for the frequency range for the filter transmission.

The nonuniform transmission response of the filter can be based on the actual transmission characteristics of the filter set for which the filter correction factor Δ_F is desired. However, a conservative and more generally applicable approach would be to base the filter transmission response upon standard requirements such as specified ANSI S1.11–1971.[16] This approach was adopted for this paper. An empirical expression was employed which predicted the filter transmission response curve identified in Fig. 4 as curve (a), the "practical" class III filter assumed for this analysis. Also shown in Fig. 4 is the minimum transmission loss required for such class III filters by the ANSI Standard (curve b) and the transmission loss of a typical analog filter (curve c) which conforms to this standard.

The "practical" filter response closely follows that of an actual filter near the filter band edge frequencies and falls slightly below the ANSI required minimum loss at frequencies well removed from the filter band edges. This "practical" filter response was generated by slightly modifying the expression in Ref. 16 for the minimum required transmission loss to obtain

$$TL(f) = 10\log_{10}[A + B[C(f/f_c) - (f_c/Cf)]^6], \quad \text{dB} \qquad (25)$$

where, for $\frac{1}{3}$-octave band filters (see Appendix E of Ref. 6 for corresponding constants for octave band filters)

$$A = 8/13,\ B = 2547,\ \text{and}\ C = 10^{-1/60}, \quad \text{for}\ f \le f_1',$$

$$A = 1,\ B,\ C = 0, \quad \text{for}\ f_1' < f < f_2',$$

$$A = 8/13,\ B = 2547,\ \text{and}\ C = 10^{1/60}, \quad \text{for}\ f \ge f_2'$$

and $f_1', f_2' = 10^{-1/30}\cdot f_c$ and $10^{1/30} f_c$, respectively, the band edge frequencies of the "practical" filter which fall slightly inside the nominal band edge frequencies f_1 and f_2 of an ideal $\frac{1}{3}$-octave band filter. Note that, outside the passband of this practical filter, the constant B in the preceding equation differs slightly from the standard value in Ref. 16 in order that the transmission loss will be zero at frequencies f_1' and f_2' and also equal to 4.5 dB at the nominal band edge frequencies f_1 and f_2, close to the value of 4.34 dB recommended by Sepmeyer for practical filters.[15]

The final expression for the filter correction factor $\Delta_F(R)$ at the receiver is formed by expressing the continuous integrals of Eq. (2) in the form of summations over each of the j discrete elements as follows:

$$\Delta_F(R) = 10\log\left[\sum_j^{nb}\right] B'_{rj} - 10\log\left[\sum_j^{b} B_{rj}\right], \quad \text{dB}, \qquad (26)$$

where B'_{rj} is the band power at the receiver in the jth integration element as defined by Eq. (9), but with the pressure spectral density P^2_{sj} at frequency f_j replaced by the filtered value $P^2_{sj}10^{-TL(f_j)/10}$. The summation for the first term above is carried out over nb segments—n bands to cover the effective transmission response range of the filter and b segments per band. The second summation term is identical to the first term in Eq. (24), where the segment powers B_{rj} are defined by Eq. (9) and are summed only over the b segments in the

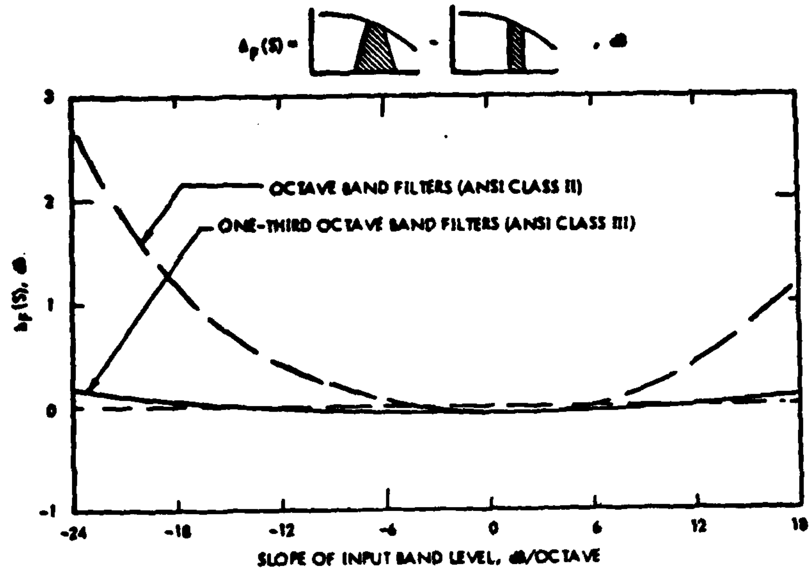


FIG. 5. Correction factor $\Delta_f(S)$ at source for various input spectrum slopes.

passband of the ideal filter.

Equations (10) or (11) for the slope parameter m_j are also modified to include the additional change in the input spectrum due to the filter response. For example, Eq. (11) becomes

$$m_j = [L_s(f_{j+1}) - TL(f_{j+1}) - L_s(f_j) + TL(f_j)]/(10/b) \times \log_{10} r \,. \qquad (27)$$

Typical results of applying this process are illustrated in the next section. Note that if B_{sj} is used instead of B_{rj} in Eq. (26), the total correction Δ, instead of only Δ_f, is computed. This procedure is frequently used when the spectral density at the source is known.

IV. RESULTS

The filter error at the source $\Delta_f(S)$, before any substantial additional spectrum shaping has occurred due to air absorption, is normally very small for input spectra with a constant slope, as illustrated in Fig. 5. In fact, for $\frac{1}{3}$-octave band filters which equal or exceed ANSI class III specifications, the filter error at the source $\Delta_f(S)$ can probably be considered completely negligible for most source spectra. However, at a distant receiver, quite a different picture emerges. Figure 6 illustrates the value of the filter error at the receiver $\Delta_f(R)$ for typical jet aircraft spectra[17] that might be observed on approach or takeoff over the propagation path lengths indicated. The figure is drawn to illustrate how the true received spectrum in such cases should actually have a very rapid cutoff at frequencies above 2000–5000 kHz. Thus, when $\Delta_f(R)$ is not accounted for in spectrum analysis of distant aircraft sounds, high frequency levels may be significantly overstated.

This is illustrated more directly in Fig. 7 which illustrates the absorption attenuation as a function of propagation path length for the same typical jet aircraft takeoff noise spectra used in Fig. 6. One-third-octave band levels that would be measured (ignoring spherical spreading losses) are shown for four frequencies with ideal or practical filters for 15 °C and 70% relative humidity. For comparison, the excess attenuation at the band (geometric) center frequency is also shown. Comparing the latter with the curves for the ideal filter illustrates the magnitude of the ideal filter correction Δ_i. Comparing the single frequency curve with the top curve in each group illustrates the magnitude of the combined correction factor $\Delta_i + \Delta_f(R)$ at the receiver. Clearly, these errors become quite large at high frequencies and large distances. *However, when these errors are large, the affected band levels are usually low and hence weighted overall noise levels may be only slightly influenced* by the errors discussed in this paper.

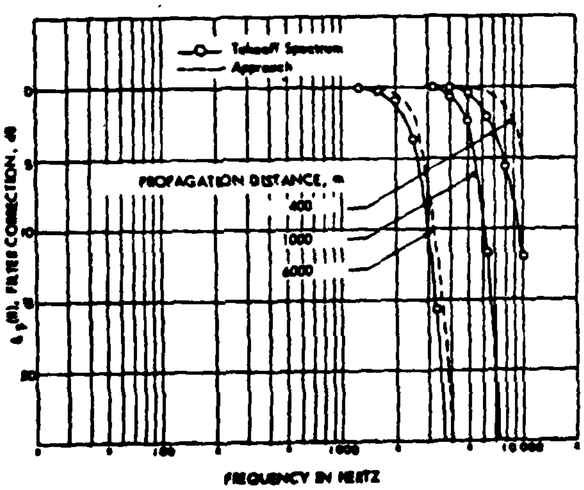


FIG. 6. Typical variation in filter correction factor $\Delta_f(R)$ at receiver for source spectra from aircraft propagated over various distances.

Figure 8 illustrates another general indication of the magnitude of the combined error in measured band levels $\Delta_i + \Delta_f(R)$ that occurs when one assumes that ab-

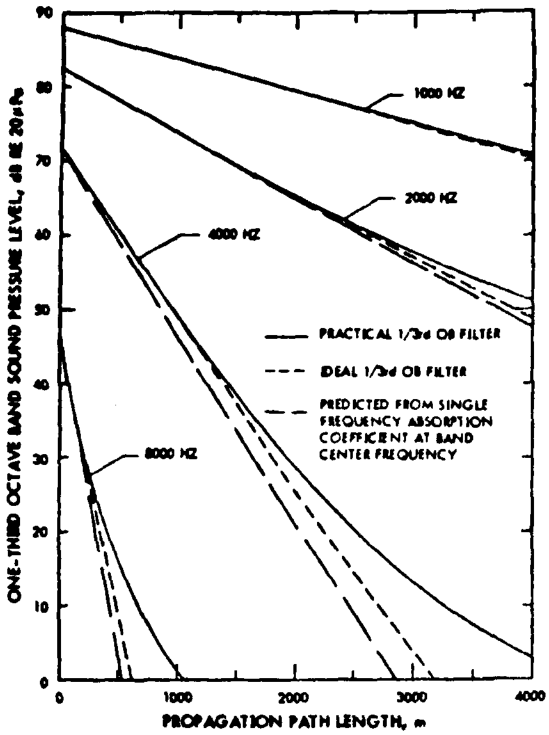


FIG. 7. Excess attenuation due to atmospheric absorption of $\frac{1}{3}$-octave band levels for typical turbofan aircraft takeoff spectrum—compairing predicted loss at band center frequency and levels that would be measured by ideal and practical filters.

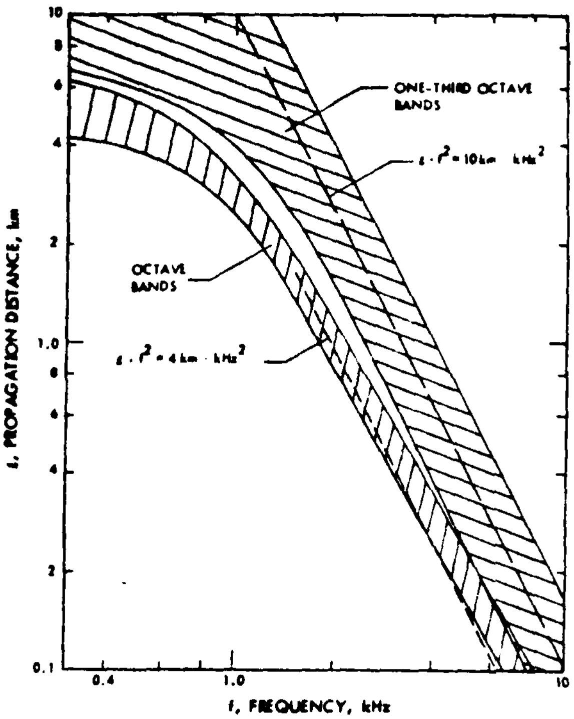


FIG. 9. Approximate boundaries of frequency and propagation distance for which total band attenuation correction exceeds 0.5 dB. An approximate rule of thumb is given by the dashed lines which indicate that the total band correction exceeds 0.5 dB when the product of propagation distance, in km, and the square of frequency, in kHz, exceeds 4 or 10 for octave and $\frac{1}{3}$-octave bands, respectively.

sorption attenuation of a band level equals that of the band center frequency. Similar to Fig. 3, this figure shows contours of the combined errors $\Delta = \Delta_i + \Delta_f(R)$ as a function of the average slope S_A of the total air absorption and assumes the source spectrum has a constant slope S_B. Off the lower right corner of Fig. 8, the combined (negative) slope of the air absorption and the source spectrum can equal or exceed the positive slope of the lower filter side band and the numerical integration does not converge. However, for real spectra, the slope of the spectra will eventually decrease sufficiently so that one always measures a finite band level.

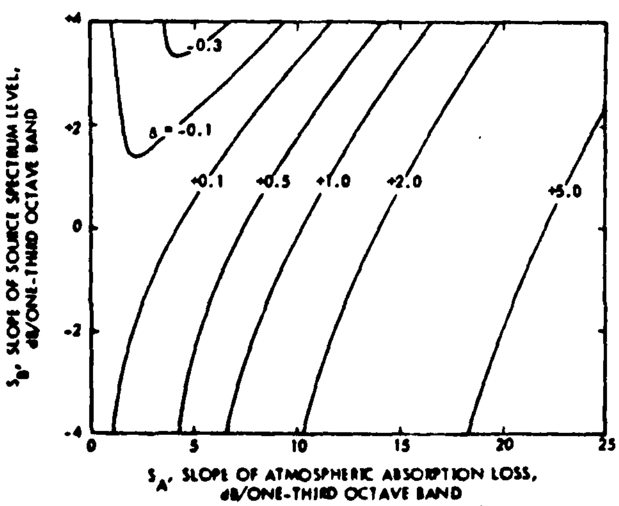


FIG. 8. Curves of constant band correction factor (Δ in dB) in terms of the slope of the source spectrum and atmospheric attenuation curve for an ANSI Class III $\frac{1}{3}$-octave band filter at 288 K and 70% relative humidity.

Figure 9 provides an approximate guideline as to the frequency–distance regimes where the combined filter error $\Delta_i + \Delta_f(R)$ will exceed 0.5 dB for the type of "practical" filters assumed for this analysis.

One final area of potential application of these correction factors for bands of noise subject to significant excess propagation losses is concerned with normalizing measured spectra from one weather condition to that of another weather condition. A related problem is concerned with the evaluation of propagation losses for bands of noise utilizing a layered atmosphere model. As a typical example of the former, assume that a spectrum is measured when the atmosphere is homogeneous, with a temperature of 25 °C, the relative humidity equals 70% and an ANSI class III $\frac{1}{3}$-octave band filter is used. For reference conditions of 15 °C, 70% relative humidity and a perfect $\frac{1}{3}$-octave band filter, at 4 kHz, and a propagation distance of 400 m, the measured band would be within a half of a dB of that which would have been measured under reference conditions. We can conclude,

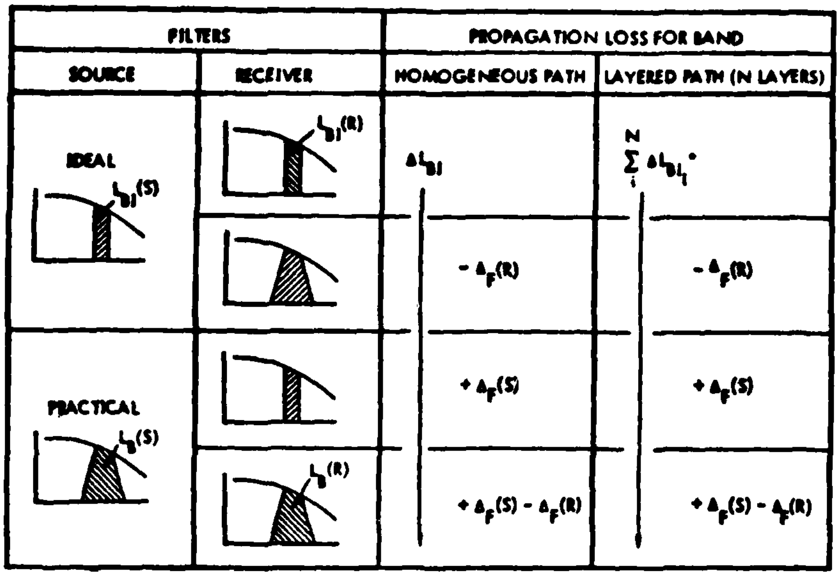


FIG. 10. Application of correction factors for computing propagation loss of bands of noise.

$$^{*}\Delta L_{BI_i} = \begin{cases} L_{BI}(i) - L_{BI}(i+1) \\ \text{or} \\ \alpha_i(f_c)\cdot \Delta\ell_i - \Delta_{I_i}(f_c, \ell_i) \end{cases}$$

$\alpha_i(f_c)$ = PURE TONE ABSORPTION LOSS COEFFICIENT FOR I-TH LAYER AND FREQUENCY f_c AT CENTER OF BAND

$\Delta\ell_i$ = LENGTH OF I-TH PATH LAYER

$\Delta_{I_i}(f_c, \ell_i)$ = [LOSS FOR PURE TONE] - [LOSS FOR IDEAL NOISE BAND] FOR I-TH LAYER

therefore, that the effect will only be significant when the propagation distance is very large (~km's), the frequency is very high (~kHz), or the actual and standard filters vary greatly.[18]

Finally, Fig. 10 summarizes the application of the two correction factors discussed in this paper. As outlined in the chart, correction for different propagation paths (or different weather conditions) is accounted for primarily by changes in only the ideal filter correction factor Δ_I. For layered atmosphere models, the filter correction factor $\Delta_F(R)$ is only applied once at the receiver. This correction term could, in fact, change when correcting measured data at a receiver from one weather condition to another since the shape of the spectrum entering the filter might be different.

The method outlined in this paper for treating the influence of the spectrum shape on atmospheric absorption of bands of noise should provide reasonably accurate results except when a pure tone is imposed on the spectrum. In the absence of a pure tone, the slope of the spectrum is typically easy to estimate to within 2 or 3 dB per bandwidth and the band absorption varies only slowly with spectrum slope. When a pure tone is present, the band containing that tone can be integrated numerically with some estimate of the pure tone frequency within the band while the remainder of the noise spectrum can be corrected for atmospheric absorption as outlined here.

V. CONCLUSIONS

The sound level of a band of noise examined with a filter after propagating through the atmosphere is a function of the atmospheric conditions, the length of the propagation path, the characteristics of the filter, and the shape of the initial noise spectrum as well as the source intensity or pressure level. For propagation at high frequencies or over long distances, the attenuation of the source band at the receiver due to atmospheric absorption is not accurately given by the attenuation of the band center frequency. For illustration, it is convenient to break the potential error into two parts, although in actual calculations, it is desirable to compute the combined correction $\Delta = \Delta_I + \Delta_F$. The first part, Δ_I, defines how the true band level measured with an ideal filter differs from the pure tone absorption computed at the band center frequency. The other factor, Δ_F, accounts for the finite transmission of practical filters outside their passband. For long distances or high frequencies, the band loss can be much smaller (or only slightly larger) than the pure tone absorption loss. Actual filter characteristics enhance this effect making it difficult *if not impossible*, in extreme cases, to deduce the true source spectrum from values observed at a distance receiver. Correcting a receiver spectrum to standard conditions is more complex but typically may involve only a small correction for band filter effects.

In view of the rather large corrections to be applied

to measured band levels to account for atmospheric absorption in extreme cases, estimates of this quantity should be made while planning a series of measurements. The propagation correction can be reduced by reducing propagation distance and selecting narrow-band filters. The fact that the band correction is largest (and sometimes not well defined) for high frequencies casts doubt on spectral measurements made of distant signals such as aircraft noise or shock waves with significant high frequency components at their source.

This paper has dealt with the correction of a source spectrum. In the real world, one has only a received spectrum. To follow the reverse path will require an iterative procedure until a source spectrum is found such that when the spectrum is corrected for atmospheric and filter effects, the measured received spectrum results. This procedure is now being investigated with particular attention to the question of whether or not the deduced source spectrum is unique. These results will be reported in a future paper.

ACKNOWLEDGMENT

The authors wish to express their appreciation to members of the ASA S1-57 Working Group on Sound Propagation for helpful discussions and criticism.

[1]C. M. Harris, "Absorption of Sound in Air Versus Humidity and Temperature," J. Acoust. Soc. Am. 40, 148-162 (1966).

[2]L. C. Sutherland, "Review of Experimental Data in Support of a Proposed New Method for Computing Atmospheric Absorption Loss," Department of Transportation Report TST-75-87, 1975.

[3]F. Douglas Shields and H. E. Bass, "Atmospheric Absorption of High Frequency Noise and Application to Fractional-Octave Bands," NASA Contractor Report CR-2760, 1977.

[4]L. B. Evans, H. E. Bass, and L. C. Sutherland, "Atmospheric Absorption of Sound: Theoretical Predictions," J. Acoust. Soc. Am. 51, 1565-1575 (1972).

[5]J. E. Piercy, "Role of the Vibrational Relaxation of Nitrogen in the Absorption of Sound in Air," J. Acoust. Soc. Am. 46, 602-604 (1969).

[6]"American National Standard Method for the Calculation of the Absorption of Sound by the Atmosphere," ANSI Standard S1.26/ASA23—1978.

[7]J. E. Piercy, T. F. W. Embleton, and L. C. Sutherland, "Review of Noise Propagation in the Atmosphere," J. Acoust. Soc. Am. 61, 1403-1418 (1977).

[8]E. H. Brown and S. F. Clifford, "On the Attenuation of Sound by Turbulence," J. Acoust. Soc. Am. 60, 788-794 (1976).

[9]A. D. Pierce, "Fundamental Nonlinear Equations of Atmospheric Acoustics: A Synthesis of Current Physical Models," J. Acoust. Soc. Am. 56, S42 (1974).

[10]D. C. Henley and G. B. Hoidale, "Attenuation and Dispersion of Acoustic Energy by Dust," J. Acoust. Soc. Am. 54, 437-445 (1973).

[11]J. E. Cole and R. A. Dobbins, "Measurement of the Attenuation of Sound by a Warm Air Fog," J. Atmos. Sci. 28, 202-209 (1971).

[12]M. A. Theobald, D. A. Webster, and D. T. Blackstock, "Outdoor Propagation of Spherically Spreading Acoustic Waves of Finite Amplitude," J. Acoust. Soc. Am. 61, S93(A) (1977).

[13]L. W. Sepmeyer, "Bandwidth Error of Symmetrical Bandpass Filters Used for Analysis of Noise and Vibration," J. Acoust. Soc. Am. 34, 1653-1657 (1962).

[14]L. W. Sepmeyer, "On the Bandwidth Error of Butterworth Bandpass Filters," J. Acoust. Soc. Am. 35, 404-405 (1963).

[15]SAE Committee A-21, "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity for Use in Evaluating Aircraft Flyover Noise," Society of Automotive Engineers, Aerospace Recommended Practice ARP-866A, issued August 1964, reissued March 1975.

[16]American National Standard S1.11-1971 (R-1966), "Specifications for Octave, Half-Octave, and Third-Octave Band Filter Sets," American National Standards Institute, New York.

[17]J. E. Mayer *et al.*, "FAAJT3D Quiet Nacelle Retrofit Feasibility Program, Volume III," The Boeing Company for Federal Aviation Administration, Report No. FAA-RD-73-131, III, February 1974.

[18]A. H. Marsh, "Atmospheric Absorption Adjustments for Bands of Noise," Report by Dytec Engineering Co., for U. S. Department of Transportation, Report No. FAA-RD-77-167, December 1977.

APPENDIX C

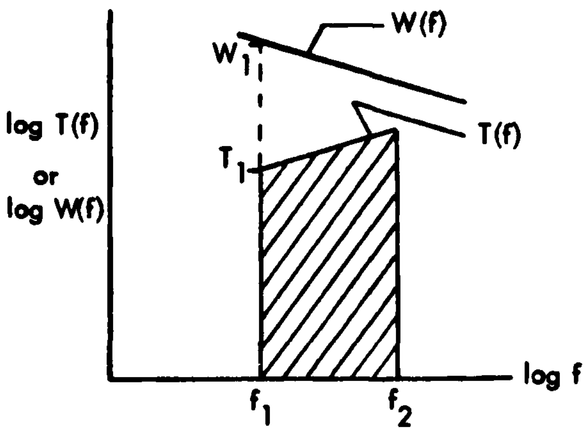
ANALYSIS OF POWER TRANSMISSION THROUGH A PRACTICAL FRACTIONAL-OCTAVE BAND FILTER

1.0 INTRODUCTION

The filter error Δ_F is defined, in the main body of the text, as the difference in decibels between the power actually transmitted by a practical filter and an ideal filter. This problem is treated in detail in this appendix for the case of various constant slope input signals. The analysis involves numerical integration over the effective transmission range of a mathematical approximation to the real transmission response of an actual filter. This approximation closely matches the actual transmission response of current technology 6-pole Butterworth filters for frequencies extending roughly between the 20 dB down points of the filter response and then tends to provide a conservative approximation to the filter response, within ANSI standard specifications,[20] for lower portions of the filter skirts. It is possible, with a digital filter technology, to provide much sharper filter skirts than is provided by a 6-pole Butterworth filter. (The latter has a roll-off rate approaching about 70 to 80 dB/octave in the steepest portions of the filter response.) However, current commercial practice is often limited to simulating, digitally, no better than 6-pole Butterworth filter response characteristics.

It should be pointed out that the specification for the minimum transmission response characteristics of one-third octave band filters to be used for aircraft noise certification are specified by the International Standard IEC Publication 225.[7] This is less stringent in roll off characteristics than that specified in the comparable American Standard S1.11[20] for frequencies where the attenuation is -20 dB or more below the pass band response. (These response characteristics are compared in Figure 1 of this appendix.) Thus, the results outlined in this appendix would be somewhat optimistic for spectrum analysis filter sets which met the IEC Standard but not the ANSI Standard. In fact, however, most commercially available filter systems meet both standards, so that these results are actually slightly pessimistic for spectra with large negative slopes.

2.0 INTEGRATED POWER TRANSMISSION



Over any bandwidth $f_1 - f_2$, the total power B transmitted by a filter with a power transmissibility function T(f) driven by a broadband noise with a spectral density W(f), as indicated in the sketch, is given by

$$B = \int_{f_1}^{f_2} T(f)\, W(f) \cdot df \qquad (1)$$

2.1 Constant Slope Case – Ideal Filter

Let the input spectrum over any band f_1 to f_2 be given in the form

$$W(f) = W_1 \cdot \left(f/f_1\right)^m \qquad (2)$$

where

$$W_1 = W(f) \text{ at } f = f_1$$

and the filter response over the same band be given by

$$T(f) = T_1 \cdot \left(f/f_1\right)^n \qquad (3)$$

where

$$T_1 = T(f) \text{ at } f = f_1$$

Then, for this constant slope case, the power (B) transmitted through the filter in this band can be defined in closed form by the integral

$$B = \frac{W_1 T_1}{f_1^{\,m+n}} \int_{f_1}^{f_2} f^{m+n} \, df = \frac{W_1 T_1}{f_1^{\,m+n}} \left[\frac{f_2^{\,m+n+1} - f_1^{\,m+n+1}}{m + n + 1} \right]$$

or

$$B = W_1 T_1 \, f_1 \left[(f_2/f_1)^{m+n+1} - 1 \right] \Big/ (m + n + 1) \tag{4}$$

2.2 General Case

For the more general case, such as for the power transmitted through an actual filter, the filter response function is definable in closed form as a smooth curve but the result is not readily integrable. A piece-wise integration is therefore necessary using the above results for each segment.

2.2.1 ANSI Filter Response

The power transmission curve for ANSI filters, considering the minimum transmission loss boundary, can be defined by an equation of the form (see Appendix A)

$$TL = 10 \log_{10} [T(f)] = -10 \log_{10} \left[a + b \left(\frac{f}{f_i} - \frac{f_i}{f} \right)^k \right] \quad , \text{dB} \tag{5}$$

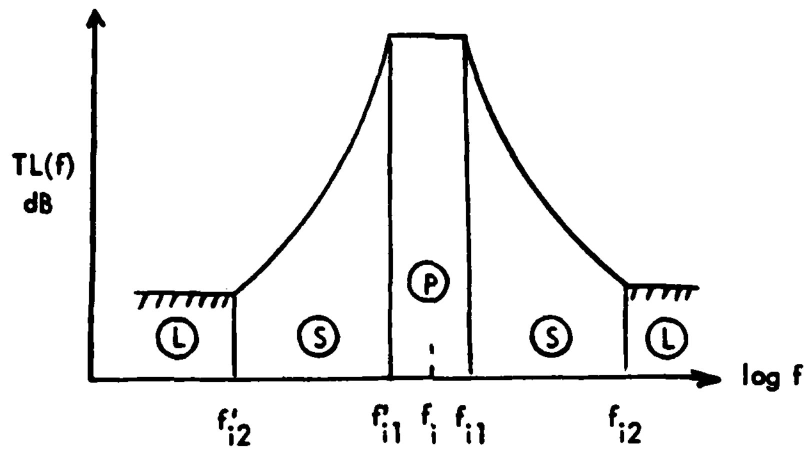
where a, b, k are constants for each portion of the filter response curve and f_i = geometrical mean frequency of the filter band.

There are three regions for the filter response curves as indicated in the sketch below.

P the "zero loss" pass band, f'_{i1} to f_{i1} where $f'_{i1}/f_i = f_i/f_{i1}$

S the "stop" bands, f'_{i2} to f'_{i1} and f_{i1} to f_{i2} where $f'_{i2}/f_i = f_i/f_{i2}$

L the limiting stop bands, $< f'_{i2}$ and $> f_{i2}$



General Shape of Frequency Response of Filter with Minimum Transmission Loss Allowed by ANSI Specification

The following table summarizes the constants a, b, k, and the frequencies, relative to f_i, for ANSI Class III one-third octave band and Class II octave band filters for each of these three regions.

Table 1

Constants for Minimum Transmission Loss Curve for ANSI Filters*

Filter Type	Frequencies ($f_i = 1$)				Region P			Region S			Region L		
	f'_{i2}	f'_{i1}	f_{i1}	f_{i2}	a	b	k	a	b	k	a	b	k
Class III 1/3rd Octave	1/5	$2^{-1/6}$	$2^{1/6}$	5	1	0	0	8/13	2500	6	$10^{7.5}$	0	0
Class II Octave	1/8	$2^{-1/2}$	$2^{1/2}$	8	1	0	0	2/3	8/3	6	10^{6}	0	0

*See Appendix C-1.

Figures 1 and 2 show the frequency response curves corresponding to these minimum attenuation requirements. Response curves for typical actual filters are also illustrated. For conservatism, it is desirable to use a response curve just slightly better than the minimum attenuation specification. This provides an approximation to a real filter without the use of any one existing model and also insures a conservative result for the filter effect.

2.2.2 "Practical" Filter Response

To approximate the "practical filter," it was convenient to simply modify Equation (5) by multiplying the frequency f by a scaling factor c which has the following values. (The exact values, used for c, equal to powers of 10, are defined along with the approximately corresponding values as powers of 2.)

$$\text{For one-third octave band filters, } c = \begin{cases} 10^{1/60} \simeq 2^{1/18} & , f > f_i \\ 10^{-1/60} \simeq 2^{-1/18} & , f < f_i \end{cases}$$

$$\text{For octave band filters, } c = \begin{cases} 10^{1/20} \simeq 2^{1/6} & , f > f_i \\ 10^{-1/20} \simeq 2^{-1/6} & , f < f_i \end{cases}$$

The "zero transmission loss" frequency bandwidths decrease accordingly so that the frequencies f'_{i1} and f_{i1} become

For 1/3rd octave band filters, $f'_{i1} = 2^{1/18} \cdot 2^{-1/6} \cdot f_i = 2^{-1/9} \cdot f_i \simeq 10^{-1/30} f_i$,

and $f_{i1} = 10^{1/30} f_i = 1.0798 f_i$

For octave band filters, $f'_{i1} = 2^{1/6} \cdot 2^{-1/2} \cdot f_i = 2^{1/3} f_i \simeq 10^{-1/10} \cdot f_i$,

and $f_{i1} = 10^{1/10} f_i = 1.2589 f_i$

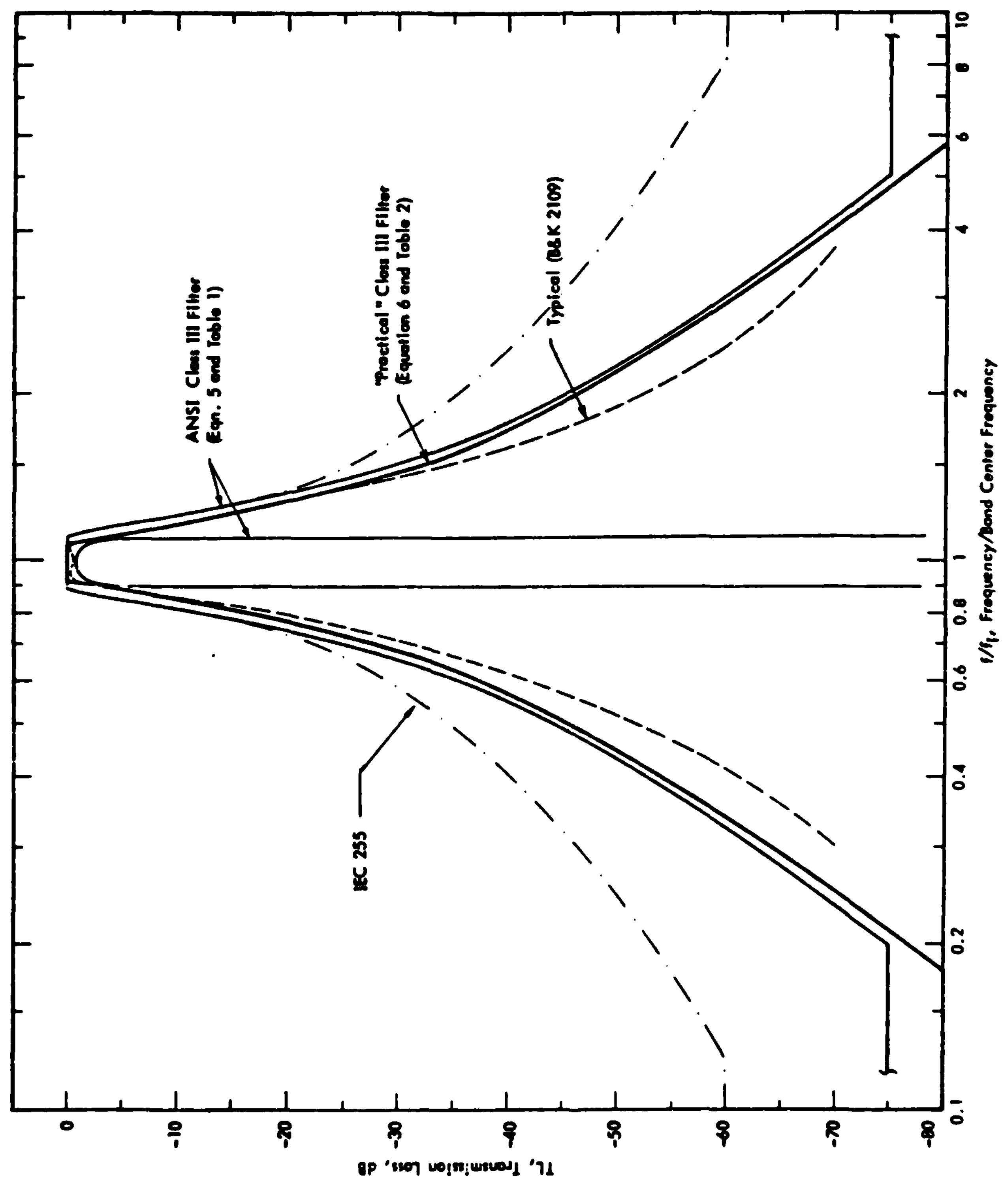


Figure 1. One-Third Octave Band Filter Response

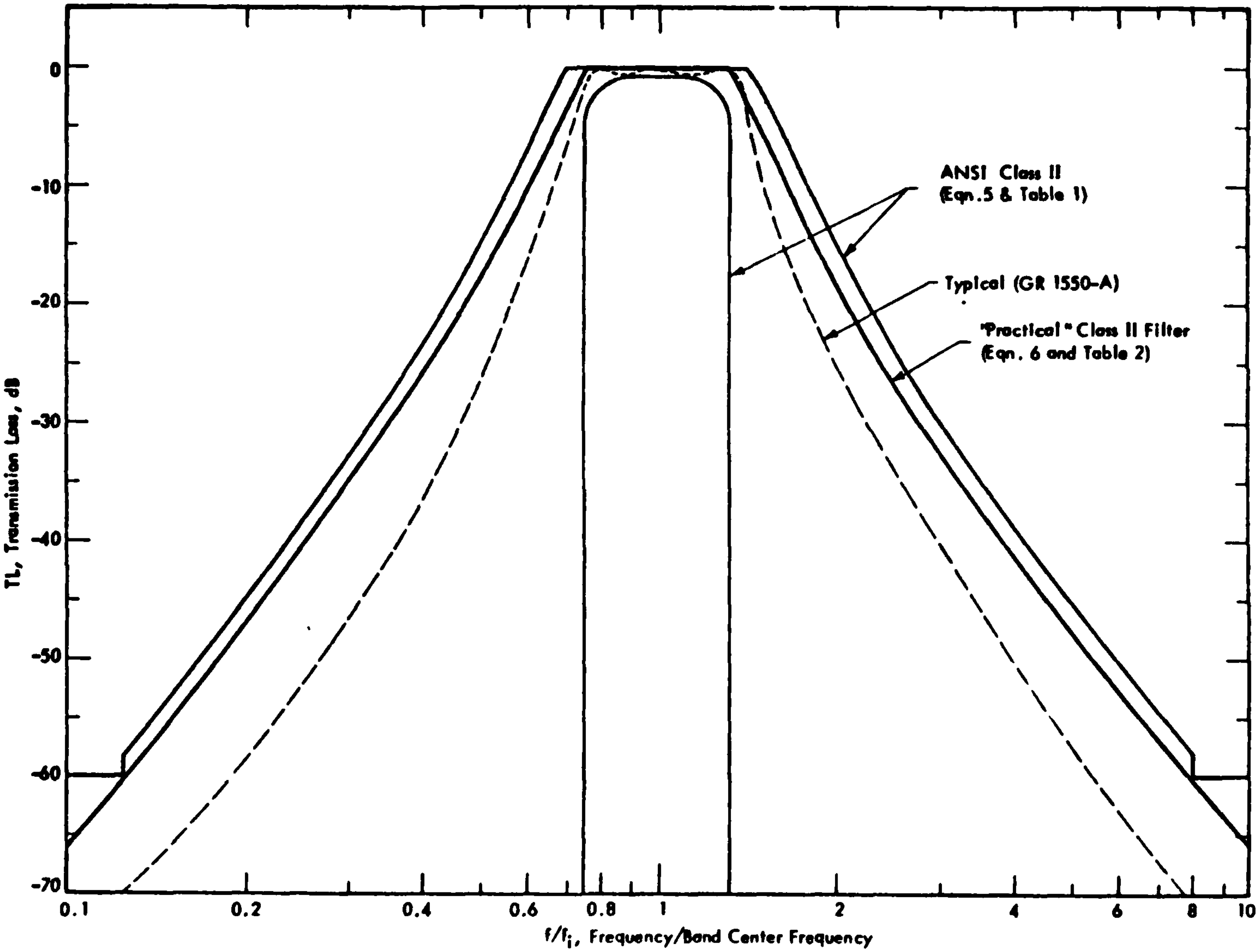


Figure 2. Octave Band Filter Response

For the "practical" filter, it was assumed that the filter response in the limiting stop bands (region L) could be extrapolated from the response in the stop bands (region S) by using the constants in Equation (5) appropriate for this region to compute the response for the limiting stop bands (region L). This extrapolation was carried out to upper and lower limiting frequencies which were set approximately equal to $10\ f_i$ and $f_i/10$. This arbitrary procedure was made to avoid unrealistic power transmission of the filter in the limiting stop bands. In actual practice, the noise floor of a spectrum analysis system will often be comparable to the power transmitted by a filter at these extreme frequency regions on either side of the pass band of a filter.

In summary, then, the "practical" filter response is computed with the following equation and parameters indicated in Table 2 for each filter type and frequency range.

$$TL = 10 \log_{10}\left[a + b\left(\frac{cf}{f_i} - \frac{f_i}{cf}\right)^6\right] \quad , \text{dB} \qquad (6)$$

Table 2

Constants for Computing Transmission Loss of "Practical" Filters Which Approximate the Minimum Transmission Loss of ANSI Filters

Filter Type	Limiting Frequencies		$1/10\ f_i < f < f'_{i1}$ Lower Stop Band			$f'_{i1} \le f \le f_{i1}$ Pass Band		$f_{i1} < f < 10\ f_i$ Upper Stop Band		
	f'_{i1}/f_i	f_{i1}/f_i	a	b*	c	a	b	a	b*	c
Class III 1/3 OB	$10^{-1/30}$	$10^{1/30}$	8/13	2547	$10^{-1/60}$	1	0	8/13	2547	$10^{1/60}$
Class II 1/1 OB	$10^{-1/10}$	$10^{1/10}$	2/3	2.724	$10^{-1/20}$	1	0	2/3	2.724	$10^{1/20}$

*The values of b differ slightly from those in Table 1 to achieve an attenuation of 0 dB for these practical filters at the limiting frequencies f'_{i1} and f_{i1}.

The resulting transmission loss curves are also shown in Figures 1 and 2 compared to the response curves for typical filters and the ANSI specifications for minimum loss. The same information is also shown in Figures 3 and 4 on linear scales for the transmission coefficient $T(f) = \log_{10}^{-1}(TL/10)$ and frequency. On this type of plot, it is easy to recognize the relative power transmitted for each type of filter according to the areas under the corresponding response curve.

2.2.3 Integration of Filtered Spectrum

To determine the power transmitted by this "practical" filter for an arbitrary input spectrum W(f), the entire frequency range will be broken up into narrow constant-percentage frequency bands with a constant ratio between the upper and lower frequencies for each segment. This frequency ratio r will be equal to

$$r = 10^{1/b} \tag{7}$$

where b = an even integer.

Within plus or minus one filter bandwidth on each side of the nominal band-edge frequencies of a filter, the constant b is set equal to a value of 240 for one-third octave band filters and 60 for octave band filters.* Outside this critical range, wider integration segments are satisfactory so that b is decreased to a minimum value of 20. Thus, for an octave band filter, power transmission was computed over 1/60th decade (~ 1/18th octave) segments for frequencies between $(1/(2\sqrt{2}))f_i$ and $(2\cdot\sqrt{2})\cdot f_i$, (i.e., ± 1 octave on other side of the pass band) and in 1/20th decade (~ 1/6th octave) segments outside this frequency range. It was convenient to use integration segments with band-edge frequencies defined in terms of powers of 10 instead of powers of 2 in order to provide a better match to one-third octave band filter center frequencies.

*An initial evaluation of integration of an arbitrary spectrum over an octave or one-third octave bandwidth indicated that numerical integration with segments narrower than 1/240th of a decade (or 1/72nd of an octave) changed the resulting integrated power by less than 0.01 dB.

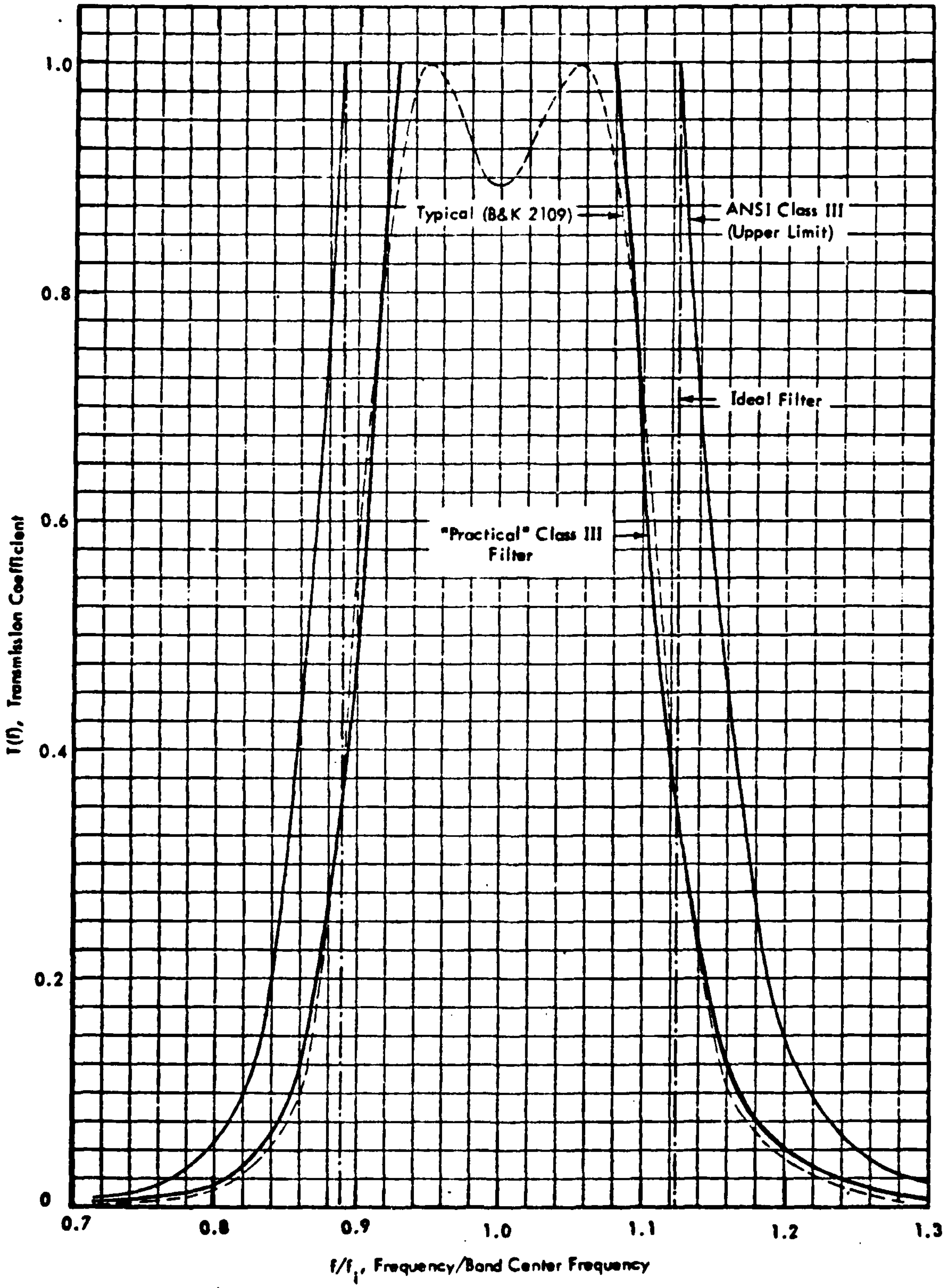


Figure 3. Relative Power Transmission for One-Third Octave Band Filter

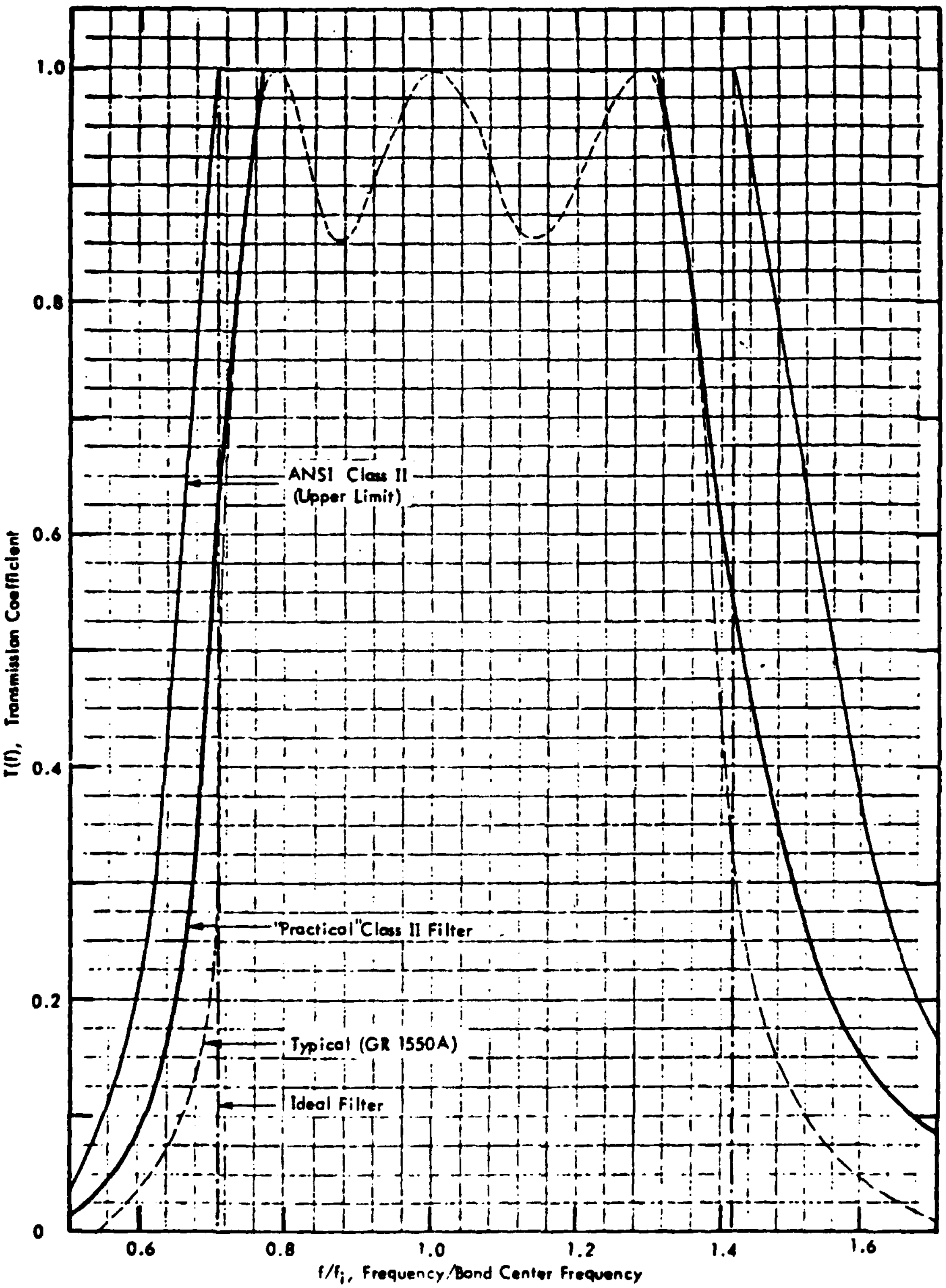


Figure 4. Relative Power Transmission for Octave Band Filters

As illustrated in Figure 5, it was assumed that, over each segment, the input spectrum and filter transmission response could be described by simple power law expressions, as given by Equations (2) and (3), corresponding to constant slopes of the spectral density W(f) and the transmission function T(f) versus frequency when these are plotted on logarithmic scales.

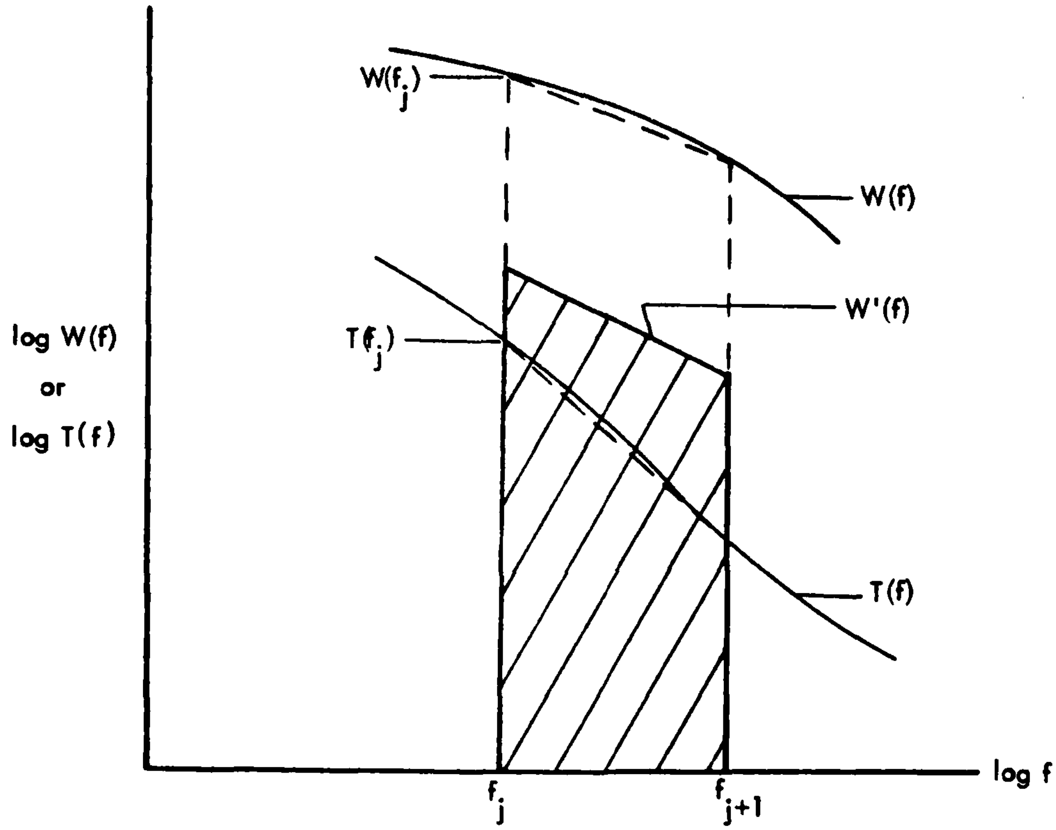


Figure 5. Illustration of Integration of Filtered Spectrum W'(f) over Frequencies f_j to f_{j+1} Using Straight Line Segments for Input Spectrum Shape W(f) and Filter Response T(f)

For the j^{th} segment, the power transmitted (B_j) can be written down, by inspection of Equation (4) as

$$B_j = W(f_j)\,T(f_j)\,f_j\left[\left(f_{j+1}/f_j\right)^{m+n+1} - 1\right]/(m+n+1) \qquad (8)^*$$

where

$W(f_j)$ = Input spectral density at frequency f_j

$T(f_j)$ = Transmissibility of filter at frequency f_j

m = Slope of input spectral density from f_j to f_{j+1}

n = Slope of filter response from f_j to f_{j+1}

For a constant slope S of the input band level spectrum in dB per bandwidth, it can be shown that the slope exponent (m) for the input spectral density is

$$m = (S/10 \log_{10} r') - 1 \qquad (9)$$

where r' = ratio of center frequencies of adjacent bands (2 or $2^{1/3}$ for octave and one-third octave filters, respectively)

The slope exponent (n) for the transmissibility function T(f) can be given by

$$n = \frac{\log_{10}\left[T\left(f_{j+1}\right)/T\left(f_j\right)\right]}{\log_{10}\left(f_{j+1}/f_j\right)}$$

or

$$n = \frac{TL\left(f_{j+1}\right) - TL\left(f_j\right)}{\log_{10}\left(f_{j+1}/f_j\right)} \qquad (10)$$

where

$TL\left(f_{j+1}\right)$ = the transmission loss of the practical filter as given by Equation (6) and Table 2 for $f = f_{j+1}$

*For the special case of $m + n + 1 \to 0$, it can be shown that Equation (8) reduces to $B_j \to W(f_j)\,T(f_j)\,f_j \ln_e\left[f_{j+1}/f_j\right]$

In the case of a more general input spectrum which does not have a constant slope, then Equation (10) can be applied to obtain the sum of the two exponents $(n + m)$ by changing the numerator to include both the input spectrum $W(f)$ and filter response $T(f)$ as follows

$$n + m = \frac{\log_{10}\left[W(f_{j+1}) \cdot T(f_{j+1}) / W(f_j) \cdot T(f_j)\right]}{\log_{10}(f_{j+1} / f_j)} \tag{11}$$

It will be convenient, in all cases, to present the resulting final integrated power through the filter in a normalized form by dividing the true power output by a reference power output for an ideal filter with a constant input spectral density equal to the value W_i at the geometric mean frequency (f_i) of the band. Thus, summing over all j segments, this relative power transmission (B_r) will be

$$B_r = \frac{\sum_j B_j}{W_i\, f_i \cdot B} \tag{12}$$

and $B = \begin{cases} 2^{-1/2} & \text{for octave band filters} \\ (2^{1/3} - 1) / 2^{1/6} & \text{for one-third octave band filters} \\ \text{The bandwidth of the ideal filter relative to its geometric mean center frequency } f_i \end{cases}$

B_j is the power transmitted in the j^{th} segment as defined by Equation (8) using Equations (9), (10), or (11) to define the exponents n and m, and Equation (6) and Table 2 to define the filter response $TL = 10 \log_{10} [T(f)]$. The input spectrum $W(f)$ is specified for the case of a constant spectrum slope by

$$W(f) = W_i \cdot \left(f/f_i\right)^m \tag{13}$$

For this case, combining Equations (8), (12), and (13), the relative power transmission is the summation

$$B_r = \sum_j T(f_j) \cdot (f_j/f_i)^{m+1} \left[(f_{j+1}/f_j)^{m+n+1} - 1 \right] \Big/ \left[m+n+1 \right] \beta \quad (14)$$

and the summation is carried out over all the j segments* necessary to encompass the entire frequency range for f_j from $f_i/10$ to $10 \cdot f_i$.

This filter response analysis has been carried out for the approximations to ANSI octave and one-third octave filters discussed earlier and for a range of input spectra with a constant slope of the true band level varying from -24 dB to +12 dB per octave. (The corresponding slope of the spectrum level is just 3 dB per octave less.) The results are presented in Table 3.

In order to provide a better picture of the power transmission through the "practical" filters, the summation involved in Equation (14) has been carried out over discrete frequency intervals corresponding to nominal bandwidths of the filter itself. Thus, in Table 3a, reading down the second column, the power transmission is shown in each of 7 octave bands, numbered -3, -2, -1, 0, 1, 2, and 3 including the center pass band (band 0), (covering a range of ±1 decade about the nominal center frequency of the filter) for an input spectrum with a band level slope of -24 dB per octave. The bottom row of the table gives the total sum from all seven of the band segments which corresponds to the total power output that would be measured by this "practical" filter. As discussed earlier, the numbers in these tables are actually the power transmitted relative to the power transmitted by an ideal filter with a white noise input having the same spectral density as the actual spectra density at the band center frequency (f_i). The bandwidth of this ideal filter is, of course, the nominal bandwidth of the actual filter (i.e., $f_i/\sqrt{2}$ for an octave filter).

A brief examination of the tables show that, as one can expect, they exhibit odd symmetry about the intersection of the 0 pass band row, and 0 db/octave column. The relative power actual transmitted through the pass band is about 0.6 dB less than for the ideal filter due to the rounding of the response curve at the band edge of the practical filter. However, for a white noise input (band slope equal +3 dB/octave), the total

*See first paragraph in Section 2.2.3, page 7, for definition of number of segments used for integration.

Table 3

A. Power Transmission for ANSI Class II Octave Band Filter - dB

REFERENCE IS 0 DB THRU IDEAL PASS BAND FILTER (BAND 0) FOR WHITE NOISE INPUT

	SLOPE OF INPUT BAND LEVEL--DB/OCT.								
BAND*	-24	-18	-12	-6	-3	0	3	6	12
-3	10.94	-7.28	-24.81	-41.74	-50.02	-58.19	-66.28	-74.31	-90.19
-2	6.00	-6.07	-17.45	-28.24	-33.47	-38.60	-43.65	-48.65	-58.49
-1	4.95	-.28	-4.91	-9.13	-11.14	-13.08	-14.98	-16.85	-20.49
0	3.25	1.69	.45	-.35	-.56	-.63	-.56	-.35	.45
1	-27.52	-24.04	-20.49	-16.85	-14.98	-13.08	-11.14	-9.13	-4.91
2	-77.77	-68.18	-58.49	-48.65	-43.65	-38.60	-33.47	-28.24	-17.45
3	-121.51	-105.91	-90.19	-74.31	-66.28	-58.19	-50.02	-41.74	-24.81
SUM	13.35	4.55	1.65	.28	-.05	-.16	-.05	.28	1.65

* NO. OF FILTER BANDS ON EITHER SIDE OF PASS BAND 0

B. Power Transmission for ANSI Class III 1/3 Octave Band Filter - dB

REFERENCE IS 0 DB THRU IDEAL PASS BAND FILTER (BAND 0) FOR WHITE NOISE INPUT

	SLOPE OF INPUT BAND LEVEL--DB/OCT.								
BAND*	-24	-18	-12	-6	-3	0	3	6	12
-10	-15.08	-35.04	-54.93	-74.75	-84.62	-94.49	-104.33	-114.16	-133.76
-9	-16.91	-34.88	-52.77	-70.59	-79.47	-88.33	-97.18	-106.01	-123.62
-8	-18.65	-34.63	-50.52	-66.34	-74.22	-82.09	-89.94	-97.77	-113.38
-7	-20.26	-34.23	-48.13	-61.95	-68.83	-75.70	-82.55	-89.38	-102.99
-6	-21.64	-33.61	-45.50	-57.32	-63.20	-69.06	-74.91	-80.74	-92.35
-5	-22.64	-32.60	-42.48	-52.29	-57.16	-62.02	-66.86	-71.69	-81.29
-4	-22.99	-30.92	-38.77	-46.55	-50.42	-54.26	-58.09	-61.91	-69.49
-3	-22.14	-28.02	-33.82	-39.55	-42.39	-45.21	-48.02	-50.81	-56.36
-2	-18.66	-22.41	-26.09	-29.70	-31.49	-33.26	-35.02	-36.77	-40.23
-1	-7.25	-8.69	-10.08	-11.44	-12.11	-12.78	-13.43	-14.08	-15.36
0	-.08	-.29	-.45	-.54	-.56	-.57	-.56	-.54	-.45
1	-17.87	-16.63	-15.36	-14.08	-13.43	-12.78	-12.11	-11.44	-10.08
2	-47.04	-43.65	-40.23	-36.77	-35.02	-33.26	-31.49	-29.70	-26.09
3	-67.31	-61.85	-56.36	-50.81	-48.02	-45.21	-42.39	-39.55	-33.82
4	-84.51	-77.03	-69.49	-61.91	-58.09	-54.26	-50.42	-46.55	-38.77
5	-100.34	-90.84	-81.29	-71.69	-66.86	-62.02	-57.16	-52.29	-42.48
6	-115.41	-103.91	-92.35	-80.74	-74.91	-69.06	-63.20	-57.32	-45.50
7	-130.06	-116.55	-102.99	-89.38	-82.55	-75.70	-68.83	-61.95	-48.13
8	-144.44	-128.94	-113.38	-97.77	-89.94	-82.09	-74.22	-66.34	-50.52
9	-158.67	-141.17	-123.62	-106.01	-97.18	-88.33	-79.47	-70.59	-52.77
10	-172.80	-153.31	-133.76	-114.16	-104.33	-94.49	-84.62	-74.75	-54.93
SUM	1.13	.42	.14	-.02	-.06	-.07	-.06	-.02	.14

* NO. OF FILTER BANDS ON EITHER SIDE OF PASS BAND 0

power transmitted for either the octave or one-third octave practical filter is only about 0.05 to 0.06 dB below that of the ideal filter.

The principal result obtained from the analysis is to clearly show the substantial portions of the total power output from a "practical" filter for an input with a constant spectrum slope that is transmitted by the filter skirts, particularly the portions of the filter response within ±1 filter band of the pass band.

Based, in part, upon these results, the spectrum iteration method defined in Section 3 of this report, was developed to make it possible, during data analysis, to account for filter response characteristics directly so that it would not be necessary to apply, operationally, the filter error term Δ_F which varies directly, in a complex way, with the signal spectrum shape, and hence, indirectly, with propagation path and weather conditions.

One other point should be clear from this appendix - the exact results obtained are uniquely defined by the filter response characteristics assumed. As shown earlier, the approximation used in this report is considered a close approximation. However, before any routine application of the spectrum iteration technique could be applied in industry, further examination of this filter response description might be desirable.

APPENDIX C-1

MINIMUM AND MAXIMUM TRANSMISSION LOSS (TL) FOR ANSI ONE-THIRD AND FULL OCTAVE BAND FILTERS

1. ANSI Third-Octave Band Filters - Class III*

(1) For $\frac{9}{10} \leq \frac{f}{f_i} \leq \frac{10}{9}$ ~ (Pass Band), the transmission loss (TL) is specified by:

$$0 \leq TL \leq 10 \log \left[\frac{5}{4} + 1300\left(\frac{f}{f_i} - \frac{f_i}{f}\right)^4\right], \text{ dB}$$

(2) For $\frac{1}{5} \leq \frac{f}{f_i} \leq 2^{-1/6}$ or $2^{1/6} \leq \frac{f}{f_i} \leq 5$ (Stop Bands)

$$TL > 10 \log \left[\frac{8}{13} + 2500\left(\frac{f}{f_i} - \frac{f_i}{f}\right)^6\right] \quad , \text{dB} \approx 0 @ \frac{f}{f_i} = 2^{1/6} \text{ or } 2^{-1/6}$$

(3) For $\frac{f}{f_i} < \frac{1}{5}$ or $\frac{f}{f_i} > 5$, (Limiting Stop Bands)

$TL \geq 75$ dB

2. ANSI Octave Band Filters - Class II*

(1) For $\frac{3}{4} \leq \frac{f}{f_i} \leq \frac{4}{3}$ ~ Pass Band Region

$$0 \leq TL \leq 10 \log \left[\frac{5}{4} + \frac{75}{2}\left(\frac{f}{f_i} - \frac{f_i}{f}\right)^6\right], \text{ dB}$$

*American National Standard Specification for Octave, Half-Octave, and Third-Octave Band Filter Sets. ANSI S1.11-1966(R-1971).

(2) For $\frac{1}{8} \leq \frac{f}{f_i} \leq 2^{1/2}$ or $2^{1/2} \leq \frac{f}{f_i} \leq 8$ (Stop Bands)

$$TL > 10 \log\left[\frac{2}{3} + \frac{8}{3}\left(\frac{f}{f_i} - \frac{f_i}{f}\right)^6\right] \quad = 0 @ \frac{f}{f_i} = 2^{1/2} \text{ or } 2^{-1/2}$$

(3) For $\frac{f}{f_i} < \frac{1}{8}$ or > 8, (Limiting Stop Bands)

$TL \geq 60$ dB